Mathematical optimization for vehicle trim in the high frequency range

S. Brandl
I. Hauer
A. Glettler
H.-H. Priebsch
ACC, Graz, Austria
T. Bartosch
MAGNA STEYR Fahrzeugtechnik AG & Co KG, Graz, Austria

sound in motion

4th Styrian Noise, Vibration & Harshness Congress
in cooperation with SAE International

NVH Excellence –
Achieving Results Beyond Customer Expectations
2006

Abstract

A main goal of the automobile industry is it to shorten the throughput time of the development process. A step towards this target is to automate time consuming optimization tasks. This paper presents an example of an automated optimization approach for the higher frequency range with the aim to minimize the sound pressure level in the car cabin.

The approach is an advancement of the Statistical Energy Analysis (SEA) theory by common mathematical optimization algorithms. As optimization variables the parameters needed for interior equipment (trim) of vehicles are defined.

To optimize acoustic packages of vehicles at higher frequencies an adequate definition of trim in SEA models is presented. It uses additional absorption, damping and insertion loss caused by a trim part as parameters for its modelling.

The presented optimization tool replaces the common trial and error approach for trim optimization by an automated and mathematically based procedure. Instead of successive testing of many different combinations of trim for the car cabin only the general requirements for the acoustic package have to be defined. These requirements can for example be dependent on weight, volume or costs. To make this algorithm applicable for the industry, also combinations of these parameters are possible.

Through principle examples the developed optimization method will be illustrated. Furthermore verification of the method, based on measurement results, will be presented using a full vehicle model. To put it in a nutshell, the integration of the presented optimization method in the car development process makes the car design more time- and cost-saving.

Introduction

Relevance of sound pressure level (SPL) predictions at higher frequencies in an early development state is steadily increasing. Therefore application of the SEA method has been increased during the last few years. Due to a statistical approach, which assumes a minimal modal overlap, the area of application for SEA is approximately limited to frequencies above 400 Hz. In this frequency range averaged SPLs and averaged surface velocities can be predicted.

Based on cost pressure and pollution minimization a main goal during the vehicle development process is to obtain a desired SPL within the vehicle cabin with minimal effort of weight and costs. This kind of optimization can easily be done for simple constructs. With increasing complexity of the construction the optimization process becomes more challenging because the overview over the effects of applied changes gets lost. Due to the immense complexity of this problem the SPL fitting process at higher frequencies nowadays is mainly based on trial and error and the profound knowledge of acoustic engineers.

With the motivation to make the optimization process more effective and more deterministic, this paper focuses on the development of a mathematically based optimization algorithm. This method allows the user to define a possible range of changes for parameters and an appropriate objective function for the problem. This information is used by an optimization algorithm for computing an optimal result for the objective function while meeting the given conditions for the optimization parameters.

Problem Formulation

To make the formulation of the optimization problem more understandable, a short introduction into SEA theory, integration of trim in SEA models and optimization algorithms is given.

SEA Theory

For application of SEA, the object of interest has to be splitted into several substructures. These substructures are called subsystems and represent parts of the body or the cavities. Therefore each subsystem is, amongst other parameters, defined by a characteristic length, area or volume, material and damping parameter.

Considering two adjacent subsystems an energy balance equation can be drawn; [8]. For this purpose the power losses within the defined subsystems and the coupling conditions between them have to be determined. After formulating the energy balance equation, calculations of the power flow between subsystems can be computed for given input powers for each of the subsystems.

Power loss within subsystems are thereby defined by the mean energy and the internal loss factor (ILF) of the subsystems. Likewise the power flow between subsystems is defined by the mean energy of the emitting subsystem and the coupling loss factor (CLF) from the emitting to the receiving subsystem. The existing power flow can be summed up by a system of linear equations that completely describes the energy behavior of the two subsystems in respect to a given excitation. This system of equations is called power balance equation (1) and can be written in matrix form, whereas the vector of the two input powers P of both subsystems is equal to the angular frequency ω multiplied by the coupling matrix *L* (matrix of loss factors) between the two subsystems and multiplied by the vector of the relevant mean energies *E*. The development of this equation and the structure of *L* are described in [8]. Negative CLFs represent the off-diagonal values of *L* whereas the ILFs are part of *L*'s diagonal.

$$\omega LE = P \qquad (1)$$

This system of linear equations can easily be expanded to any number of subsystems. Throughout this paper we assume that the number of subsystems is *n*. Then the power balance equation can be written as in (1) where *E* and *P* are *n*-dimensional vectors, ω is the angular frequency and *L* is an *n* x *n* dimensional matrix.

Using (1) SEA allows the calculation of energies, velocities and SPL for all subsystems for given frequency bands. Usually results are calculated for 1/3rd octave bands.

Integration of Trim

To optimize acoustical packages of vehicles using SEA an appropriate integration of trim into SEA models is needed. Well known for trim modeling is the BIOT theory introduced in [1]. It describes trim as a composition of different layers where each of the layers is specified by a list of parameters.

From the optimization point of view this kind of trim modeling is unfavorable due to an unknown amount of parameters for each trim part mainly depending on the number of layers for each trim part. This would lead to a different handling for each trim part and therefore makes optimization much more difficult.

For this reason trim is modeled as influence on internal and coupling loss factors. This effect is calculated from absorption, insertion loss and additional damping of each trim part as used in [2]. Through this definition each trim part is represented by the same number and type of parameters and can therefore be handled more easily in the optimization process.

Applying this kind of integration the loss factor matrix *L* becomes dependent on the optimization parameters *x*. For advantages in the calculation *L* could be split up into a constant part $L_{default}$ and a part L_{update} which is dependent on *x*. (2) As consequence the energy of the subsystems is influenced by the optimization variables through the usage of *L* in the power balance equation.

$$L(x) = L_{default} + L_{update}(x) \qquad (2)$$

Optimization

To formulate an optimization problem the objective function *f(x)*, that has to be minimized by the algorithm, and the constraints c_i, that have to be fulfilled for each solution, have to be defined; see (3). Used constraints in this optimization problem are mainly upper and lower bounds for the defined optimization parameters. For mathematical reason constraints are divided into equality and inequality constraints.

$$\boldsymbol{min}\ f(x) \text{ subject to } \begin{cases} c_i(x) = 0 & i \in E \\ c_i(x) \geq 0 & i \in I \end{cases} \qquad (3)$$

E…Index set for equality constraints
I…Index set for inequality constraints

While all given optimization parameters and their possible ranges span the parameter space, the objective function defines a cost value for each vector in the parameter space.

Formulating the objective function a variety of variables can be taken into account. First of all the SPL in the target cavity has to be considered. This is done by including the energy in the formulation of the objective function. A common solution for this integration is a squared difference of the energy at the current iteration step and the energy desired in this subsystem as shown in (4).

Through consideration of the energy the optimization variable value of the current optimization step has only an indirect influence on the objective function via the loss factor matrix and the power balance equation. Therefore values of the optimization variables should additionally be taken into account via a penalty function *p*. Thus a direct influence of the optimization parameters on the objective function can be embedded.

$$f(x) = \sum_i w_{E\,i}\left(E_i - E_{i_{desired}}\right)^2 + \sum_i w_{p\,i} \cdot p_i(x_i), \quad \text{where } E(x) = \frac{1}{\omega} L(x)^{-1} P \qquad (4)$$

This direct influence is of course depending on the type of the parameter. For each parameter type criteria are needed to define the penalty for a specific increase of the parameter.

Using Delany Bazely model the interrelationship between the absorption of a trim part and its thickness can be described; see [4] and [10]. Knowing the influence of a change in thickness of a specific trim part on its absorption an appropriate penalty function can be defined. Penalty for an increase in absorption can therefore be described by using parameters like volume or mass. Additionally an estimation of costs can also be taken into account.

Likewise mathematical models for the connection between thickness and insertion loss can be used to define the penalty function for a specific change of the insertion loss parameter values.

Additional damping through trim parts most often is caused by an increase in mass. Therefore penalty for mass increase can be best done by considering this parameter in the penalty function.

As stated in (4) weighting parameters have to be introduced to compensate the imbalance between the energy part and the penalty function. A possible solution is a scaling of the energy part and penalty function between 0 and 1. Additional adjustment can be done by introducing weighting factors $w_{p\ I}$ and weighting functions $w_{E\ i}$ that indicate the importance of specific subsystem energies and penalty values.

For calculation of the gradient of *f* with respect to *x* the chain rule has to be applied. In its application the gradient of the energy vector with respect to *x* is needed. Because of the indirect influence of the optimization parameters on the energies of the subsystems over the loss factor matrix the calculation of the gradient of the energy vector in respect to *x* is not straight forward. Applicable methods to solve this problem are given in [9].

Optimization Solver

Having formulated the optimization problem, an appropriate optimization algorithm has to be applied. In recent years application of evolutionary algorithms such as simulated annealing or genetic algorithms became very popular, see e.g. [7];

They benefit from the fact that no deeper knowledge of optimization is needed to apply these kind of algorithms to optimization problems. These optimization solvers however have the drawback that a huge amount of calculations is needed. Furthermore they are not deterministic, which leads to the fact that for two optimization runs for the same optimization problem different solutions could be calculated.

For this reason the deterministic Sequential Quadratic Programming (SQP) algorithm has been used as solver for the defined optimization problem. A detailed description of SQP can be found in [9]. To show the approach of the algorithm for solving the optimization problem a brief description of the method is given.

The only interesting point in describing optimization algorithms is how they determine the way from a starting point to the optimum. To find the next step for the optimization from the current point, this method simplifies the problem by approximating it by a quadratic problem. SQP then uses this simplified and more easily solvable model to compute the starting point for the next iteration.

Examples

For verification of the presented optimization tool it has been applied on several SEA models for the 1000 Hz 1/3rd octave band. In this paper at first a principle example will be used to illustrate the correctness and the potential of the developed approach. Furthermore verification of the method will be presented using a full vehicle model.

The used principle example is a rough model of an SUV vehicle. It consists of ten steel plates and two separated cavities with an excitation in the engine compartment.

Objective Function Formulation

As stated before the main challenge in applying a mathematical optimization is the formulation of the objective function. Results of research in this field are discussed in the following.

The easiest way to formulate the objective function is to consider only the energy of the subsystems. This allows an optimization towards a specific energy level within each subsystem. A possible objective function is given in (5).

It uses the difference of the current energy to a desired energy in one specific subsystem. In the case of acoustic optimization this subsystem is usually the car interior. An example of a surface for two absorption optimization variables using this function is displayed in Figure 2. It can been seen that the solution for the posed problem is not unique. This is very common due to the fact that the desired energy can be reached by different vectors of the parameter space of the two optimization variables.

$$f(x) = \left(E_i - E_{i_{desired}}\right)^2 \qquad (5)$$

At different locations in the vehicle interior (e.g. floor and ceiling) different composite materials, combining layers of several materials, are applied. Absorption of a trim part depends on this layer composition. Through application of (5) a change in layer composition can only be considered through a change in absorption. The associated increase in weight, volume or costs however cannot be taken into account.

One possible solution to this problem is the consideration of additional terms in the objective function that are directly dependent on current values of the optimization parameters. One possibility is the addition of penalty functions as presented in (4) and (6).

$$f(x) = w_E\, \boldsymbol{m}_E\left(\left(E_i - E_{i_{desired}}\right)^2\right) + w_p m_{\boldsymbol{p}}\, p(x) \qquad (6)$$

One example where a direct influence of the optimization variable values is beneficial is the consideration of trim part thickness. It can be determined from the layer composition of the trim part and the current absorption by application of the BIOT theory introduced in [1]. Using the trim part thickness also factors like weight and costs that are dependent on the thickness can be considered in the optimization.

Because domains of penalty functions and the energy weighting are differing, this leads to the problem of balancing the energy and the penalty part of the objective function. Otherwise the optimization would only be dependent on the part with the bigger influence on the objective function. In order to guarantee this balancing a mapping function *m* for each objective function part to the interval between 0 and 1 has been used.

In the demonstrated example, due to presentability of the objective function, two parameters are considered. Absorption on the firewall and absorption of the floor, both in the car interior. The used penalty function contains a linear approximation of the connection between thickness and absorption of the corresponding parts. This approximated thickness is used for calculation of the volume for each trim part.

The surface of the objective function of this simple example is presented in Figure 3. In comparison to Figure 2 it can be seen, that through adding of this simple penalty the solution for the optimization problem became unique.

Mapping of all objective function parts to the interval between 0 and 1 additionally allows a weighting of the importance of energy and penalty parts for the optimization. Through this weighting different types and classes of vehicles can be considered in the optimization process.

A high importance to acoustical comfort can be represented through a higher weighting of energy terms (wE) in relation to the penalty terms (wP) that are representing weights and costs. This leads to the fact that the priority of the optimization is an reduction of energy although influences of the penalty functions are considered.

Therefore this weighting expresses for example the situation that in higher quality cars more expensive and more weighty materials are used in order to reach certain SPL limits.

Results for an optimization where the acoustic comfort is considered by formulating the objective function can be seen in Table 1. In this example the absorption on the floor and on the firewall of the principle example have been optimized. For this artificial example it has been assumed that an increase of absorption for the trim part used for the floor is cheaper than an increase of absorption on the firewall.

Used penalty functions for this simple example are given in (7). The considered parameters $t_{Firewall}$ and t_{Floor} represent the thickness of the trim corresponding parts. It can be determined by application of the BIOT theory introduced in [1]. Using the surface areas of the firewall (1.36 m^2) and the floor (5.1 m^2) given penalty functions represent the consideration of the volume of the trim parts in the objective function.

$$\begin{aligned} p_{Fireall}(x) &= t_{Firewall}(x) * 5.1 \\ p_{Floor}(x) &= t_{Floor}(x) * 1.36 \end{aligned} \qquad (7)$$

Due to a larger area of the floor the optimization algorithm increases the absorption of this trim part to reduce the energy in the interior for low and medium acoustic comfort importance. If the floor absorption reaches its upper bound (0.8) the high acoustic importance demands an additional decrease in SPL. Therefore the optimization increases the absorption of the firewall to further decrease the energy in the interior.

Acoustic comfort importance	Low $w_E/w_p=1$	Medium $w_E/w_p=1.33$	High $w_E/w_p=20$
Absorption floor	0.27	0.46	0.8
Absorption firewall	0.2	0.2	0.3

Table 1: Different results for different importance of acoustical comfort using a principle example.

Although this example is very simple one can clearly see that the influence of the energy (resp. the SPL or velocity) and the penalty can be controlled by the weighting factors.

The presented examples should demonstrate some capabilities of the developed method. For these simple examples optimization could also be done manually. With an increasing number of optimization variables and an increasing complexity of the used SEA model, the application of a mathematical optimization becomes necessary to find an optimal solution.

To verify the developed method on a practical experiment the SEA model shown on the right side of Figure 1 has been used. It was established at ACC in cooperation with MAGNA STEYR Fahrzeugtechnik and was validated by measurements; [5]. As excitation an SPL in the engine compartment and the concurrently acceleration at the cantilevers measured in operational conditions has been used.

As optimization parameters absorption values on the floor, the firewall, the roof and the doors have been used. For verification of the method all areas have been equipped with the same artificial material.

Goal of the optimization was a decrease of SPL in the vehicle interior under consideration of a predefined effort for changes of absorption for the different trim parts.

For this example it was assumed that a change in absorption for the doors is most expensive. Changes in floor and firewall absorption are less complex and the least effort is an adoption of the absorption on the roof.

Additionally the weighting between energy and penalty functions has been taken into account to consider the importance of acoustic targets in relation to weight and costs. Results for the defined optimization problem are given in Table 2.

As expected it can be seen that in the solution of the optimization mostly absorption at the roof is increased. Additionally the absorption of floor and firewall are increased slightly.

Analyzing the solution it can be concluded that an exclusive increase of the absorption at the roof does not lead to an optimal solution to the posed problem when weight, costs or installation depth are taken into account.

Acoustic comfort importance	Low $w_E/w_p=5$	Medium $w_E/w_p=20$	High $w_E/w_p=50$
Absorption floor	0.49	0.52	0.53
Absorption firewall	0.50	0.54	0.56
Absorption roof	0.54	0.61	0.67
Absorption door	0.48	0.49	0.50

Table 2: Different optimization results with different importance of acoustical comfort using an SEA vehicle model.

Presented results have been computed for a single 1/3rd octave band. For application of the presented method over the whole frequency range of the SEA model various procedures have been worked out. Focus of current developments lies on the integration of these enhancements. Further applications of the presented method are planned for the near future.

Development Process Integration

For a possible integration of the presented methodology in the concept phase of the vehicle development process following four steps can be described.

1. Basic sensitivity analysis
2. First optimization (rough results)
3. Appropriate solutions offered by supplier
4. Refined Optimization for specification of final requirements

For the first step an SEA model is required. Using a FEM/SEA hybrid method [6] an SEA model can be established early in the development process as soon as the first FE model is available. For a rough estimation even an SEA model based on a CAD model would be sufficient.

Using this model the sensitivity analysis presented in [3] can be applied to determine the most important properties of trim parts for the SPL in the interior. Based on these results weighting factors w_p for the optimization can be derived.

Using acoustic targets that depend on the class and type of the vehicle and considering the results of the sensitivity analysis a rough formulation of the optimization problem can be developed in the second step. In this early state of development also experiences of acoustic engineers and former projects can be integrated to formulate weighting (w_E) and penalty (p) functions.

Based on the solution of this optimization problem calculated in the second step a rough specification of trim part properties for first discussions with suppliers can be developed. This discussions result in properties, weights and costs of appropriate solutions the suppliers offer.

Having a more detailed specification of possible composite materials for each trim part and additional input from the supplier about roughly estimated weight and costs a refinement of w_E and p in the objective function can be designed.

In the final step the solution for this detailed optimization problem can be used to define appropriate specifications for each trim part to get an optimal behavior of the car interior.

Conclusion

Target was an optimization of trim parts using SEA models. Therefore an adequate integration of trim has been chosen and an optimization tool has been developed. As solver for the optimization problem the deterministic SQP algorithm has been used.

An appropriate formulation of the objective function has been shown to be fundamental for achieving good results. Therefore this function guarantees the adaptability of the method. Depending on objectives in the development process or restrictions for specific vehicle classes the formulation of the objective function could be customized.

This leads to a broad flexibility for applications. The more data is available the more precise the optimization problem can be formulated and the more precise the results will become. To overcome problems with missing data from measurement, simulation models have been used to replace measured data by simulated values.

Even if there are no simulation models available the optimization method can be used for trend analyses. In this case the objective function can be modeled based on the experience of acoustic engineers. Due to its high adaptability the presented method is a good enhancement for nearly all stages of the vehicle development process.

Acknowledgements

Mathematical research is supported by Prof. Laback of the Institute for Mathematics at TU Graz and Prof. Volkwein of the Institute of Mathematics and Scientific Computing at the University of Graz. The research of ACC is carried out with the support and funding of MAGNA STEYR Fahrzeugtechnik. Furthermore, the project is funded by the Austrian government, the government of Styria, the Styrian Economy Support (SFG) and the city of Graz.

References

[1] Allard J. F.; Propagation of Sound in Porous Media: Modelling Sound Absorbing Materials; Chapman & Hall (1994)

[2] AutoSEA2 2005; User's Guide; ESI Group (2004)

[3] Brandl S., Hauer I., Priebsch H.-H., Bartosch T., Volkwein S.; Analysis and Assessment of the Sensitivity of Trim Parameters on SEA Simulation for Interior Noise Reduction; Proceedings ICSV13, Vienna, 2006

[4] Delany M.A., Bazley E.N., "Acoustic properties of fibrous absorbent materials", Appl. Acoustics 3, 105-116

[5] Glettler A., Hauer I., Priebsch H.-H., Bartosch T.; SEA Model Verification and Sensitivity Analysis of a Passenger Car; EuroPAM 2004, Paris

[6] Hauer I., Jalics J., Priebsch H.-H., Bartosch T., Prediction of Vehicle Interior Noise in High Frequency Range using Statistical Energy Analysis Hybrid Method, Proceedings of the Joint Congress CFA/DAGA 2004, 2004

[7] Korte S., Optimization of a car floor damping based on a complete AutoSEA car model using a genetic optimization algorithm, Proceedings of the 2nd Vibro-Acoustics User Conference Europe; Paris, 2004

[8] Lyon R., DeJong R.; Theory and Application of Statistical Energy Analysis, 2nd Edition; Butterworth-Heinemann; Newton (1995)

[9] Nocedal J., Wright S.J., Numerical Optimization, Springer, New York (1999)

[10] Schwingenschlögl M., Mathematische Modelle der Zusammenhänge zwischen physikalischen Gesetzen und dem akustischen Verhalten von Werkstoffen in der Fahrzeugindustrie, TU Graz (to be published)

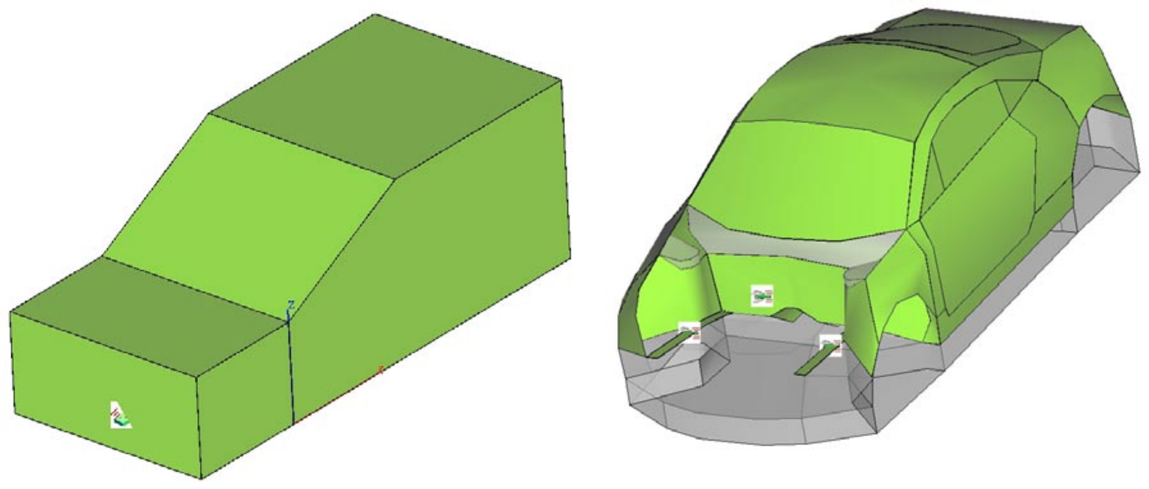

Fig. 1: SEA models used for demonstrative examples
(left: principle example; right: Audi TT)

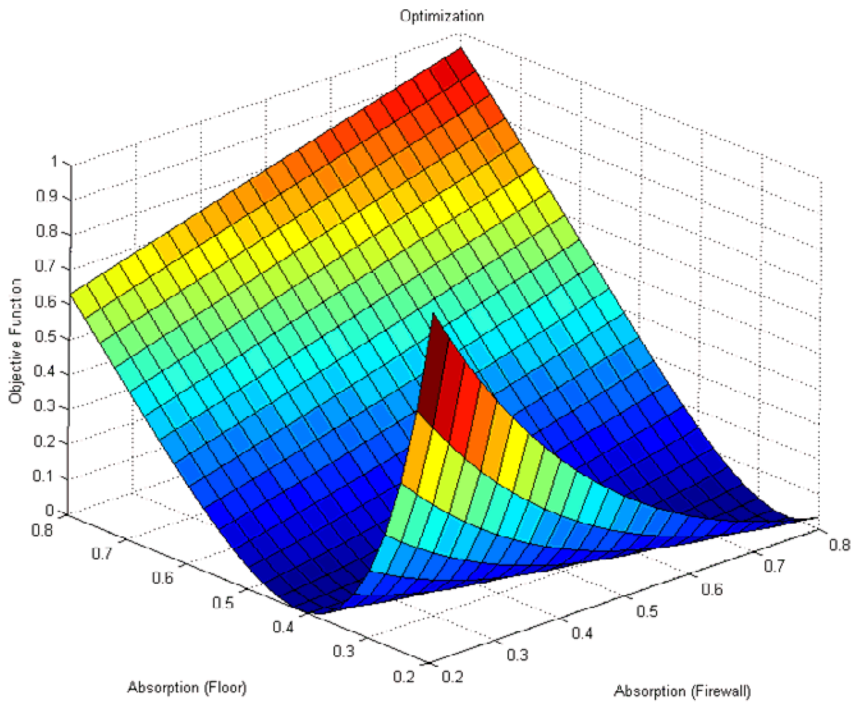

Fig. 2: Surface of objective function for two variables using function given in (5)

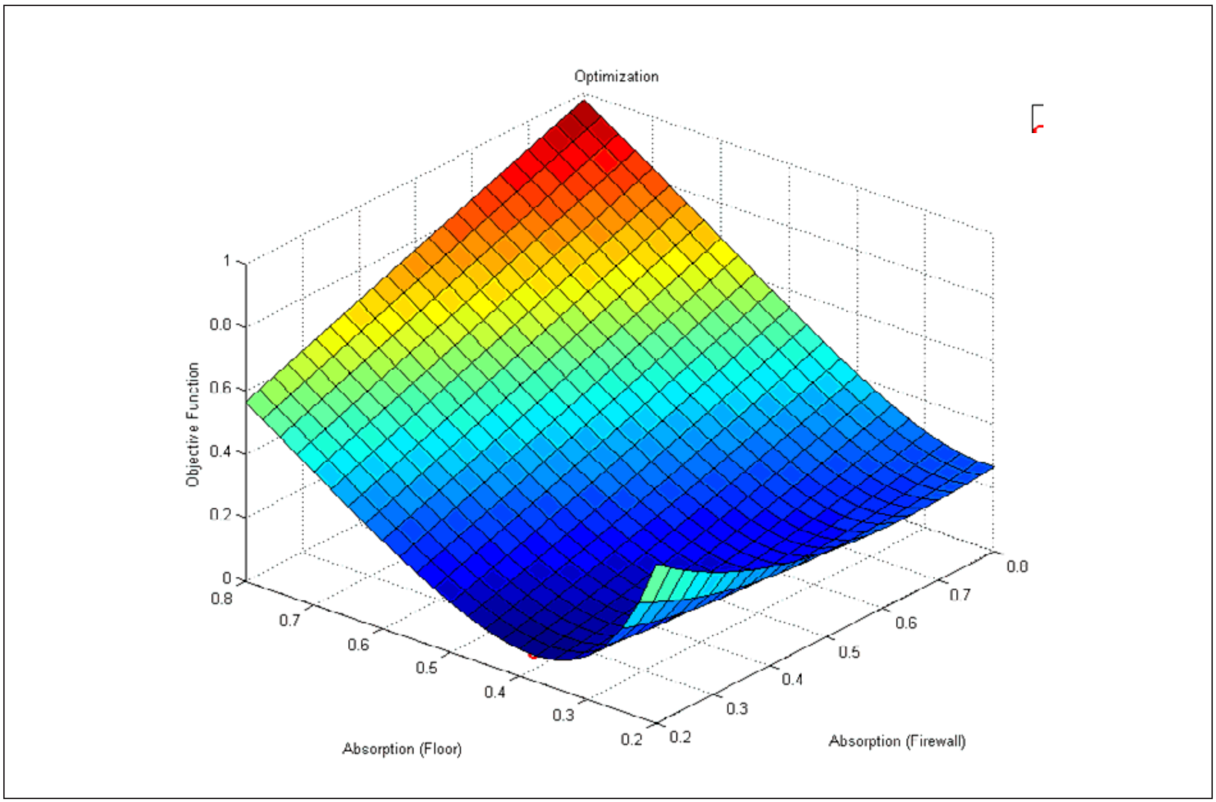

Fig. 3: Surface of objective function for two variables using function given in (6)