PAMM · Proc. Appl. Math. Mech. **4**, 179–180 (2004) / **DOI** 10.1002/pamm.200410071

Influence of Inertia on the Microscale in Homogenization

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When calculating effective dynamical properties of a material, inertia on the microscale is usually neglected. Here, contrary to these approaches, inertia effects are taken into account, leading to a frequency dependent microscopic behavior. Thus, a frequency dependent macroscopic constitutive equation is required. Therefore, a viscoelastic constitutive equation is applied on the macroscale. The material parameters are found using an Evolutionary Strategy. In the 1-D case, system responses on the micro- and macroscale show a good agreement in a frequency range from 0 up to the first eigenfrequency of the microstructure.

1 Introduction

Nearly all common materials exhibit a certain heterogeneous microstructure. The determination of macroscopic, effective properties of such microstructured materials is referred to as homogenization. There are many different approaches to calculate the effective mechanical properties of a material. A comprehensive overview is given in [1] or [2]. Most publications on homogenization, however, only concentrate on the calculation of statically loaded microstructures where in some cases an analytical solution of the problem is possible.

It is assumed that the structures to be considered are made up of identical unit cells. When assuming periodicity of the microstructure, no effort has to be put on the choice of a Representative Volume Element (RVE) because the smallest repetitive unit of the material is representative. A special interest lies on auxetic materials, i.e., materials that exhibit negative Poissons ratios. The microstructure of such a material usually has re-entrant corners. The calculation of the unit cell is performed in frequency domain due to a harmonic excitation. A frequency domain calculation is preferred against a time domain calculation because periodic boundary conditions can easily be applied. Moreover, the Boundary Element Method is used because it provides analytical exact results [3].

Contrary to other approaches, inertia effects on the microscale are taken into account. Thus, in a frequency domain calculation, the response of the unit cell is frequency-dependent which requires the macroscopic constitutive equation also to be frequency-dependent. Here, the viscoelastic constitutive equation

$$\mathcal{F}\{\sigma\} \sum_{k=0}^{N} p_k (i\omega)^k = \mathcal{F}\{\epsilon\} \sum_{k=0}^{M} q_k (i\omega)^k \tag{1}$$

is chosen (1-D case). In (1), $\mathcal{F}\{\ \}$ denotes the Fourier transformation, σ and ϵ denote the stress and strain and p_k and q_k are the viscoelastic material parameters [4]. The number of parameters N and M depends on the application. Furthermore, N and M in (1) are chosen as a whole number parameters, i.e., no use of fractional derivatives is made until now.

2 Homogenization using an Evolutionary Strategy

The homogenization is formulated as an optimization problem, i.e., 'Find material parameters on the macroscale which describe the micromechanical behavior as good as possible'. The consideration of dynamic effects on the microscale makes it necessary to use an optimization algorithm. As optimization function, the square difference between the microscopic and macroscopic system response is used. The differences are summed up over the considered frequency range. If a harmonic stress is applied on the unit cell, the minimizaion function reads

$$f = \sum_{\omega=0}^{\omega_{max}} \left(\epsilon^{micro}(\omega) - \epsilon^{macro}(\omega, p_k, q_k)\right)^2 \to 0 \quad . \tag{2}$$

In (2), $\epsilon^{micro}(\omega)$ is calculated from the unit cell, whereas $\epsilon^{macro}(\omega)$ is obtained from the macroscopic constitutive equation which contains the unknown material parameters.

An Evolutionary Strategy is used to minimize (2). The advantage of this procedure is that local minima do not cause the optimization to fail. In the present application, one gene of the Evolutionary Stategy corresponds to one material parameter p_k or q_k of the macroscopic constitutive equation. The values of different parameter sets of p_k, q_k ('individuals') are modified

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('mutated') and recombined in one iteration step ('generation') of the algorithm (for details on Evolutionary Stategies, see [5]).

3 Numerical example

The prescribed procedure is applied on the auxetic cell depicted in Fig. 1. See also [6] for an extended version of this example. The cell consists of steel beams with a quadratic cross section of $A = 10^{-8}m^2$ and a Young's modulus of $E = 2.1 \cdot 10^{11} N/m^2$. The frequency response for an applied load of $F = 100kN$ is calculated over the freqency range including the

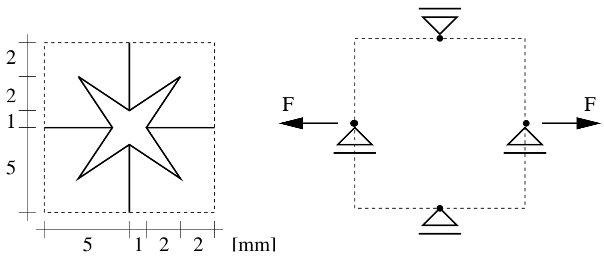

Fig. 1 Unit cell geometry and boundary conditions

first eigenfrequency of the system. Due to the symmetry of the system, the periodicity of the boundary condtions is fulfilled. In this example, a number of $p_k = q_k = 7$ material parameters is chosen. Since in the Evolutionary Strategy the starting population is chosen randomly, three optimization runs were performed. Fig. 2 shows the results of the optimization. On the left hand side the history of three performed optimization runs are plotted. The fitness value f is plotted over the number of performed iterations. On the right hand side the corresponding results are given. The strain of the system is plotted over the considered frequency range. As can be seen, the Evolutionary Strategy has found appropriate material parameters in all three

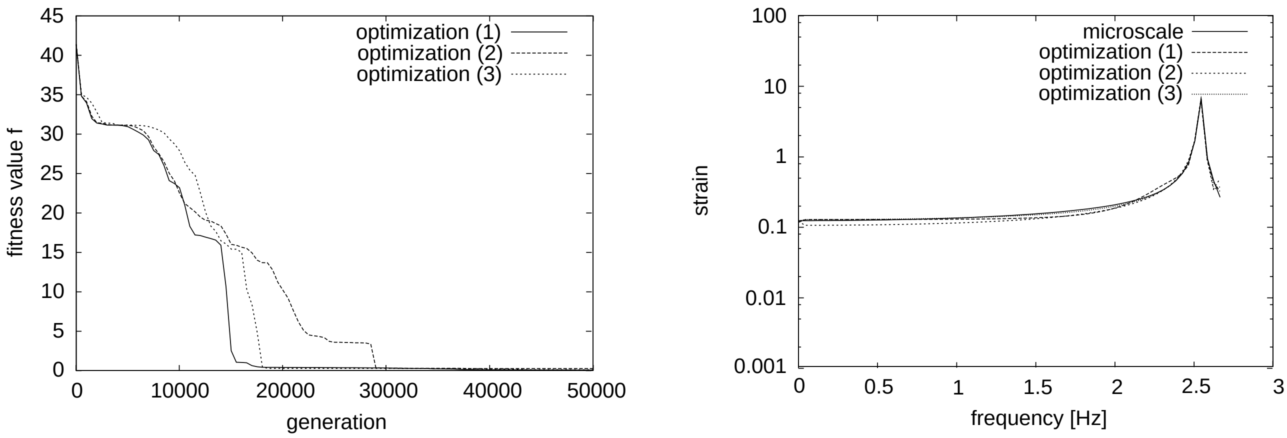

Fig. 2 Homogenization using an Evolutionary Strategy: Fitness value versus generation and resulting strain versus frequency for different optimization runs

optimization runs. The choice of an Evolutionary Strategy is well justified because a gradient based optimization procedure may fail due to the existence of local minima. Further research on the proposed homogenization method will include a larger frequency range and a generalization for the 2-D case.

References

[1] S. Nemat-Nasser and M. Hori, Micromechanics: Overall Properties of Heterogeneous Materials, first ed. (North-Holland Publication, Amsterdam, 1993).
[2] Z. Hashin, Analysis of composite materials - a survey, J. Appl. Mech. Trans. ASME 50, 481-504 (1983).
[3] H. Antes, M. Schanz and S. Alvermann, Dynamic analyses of frames by integral equations for bars and Timoshenko beams, J. Sound. Vib. (in press, 2004).
[4] R.M. Christensen, Theory of Viscoelasticity - an Introduction, first ed. (Academic Press, New York, 1971)
[5] E. Schöneburg, Genetische Algorithmen und Evolutionsstrategien, first ed. (Addison-Wesley, Bonn, 1996).
[6] S. Alvermann and M. Schanz, Numerical Homogenization of Auxetic Materials with a Vibrating Microstructure, Computational Materials Science (submitted, 2004).