Thin Layer Transition Matrix description applied to the Finite Element Method

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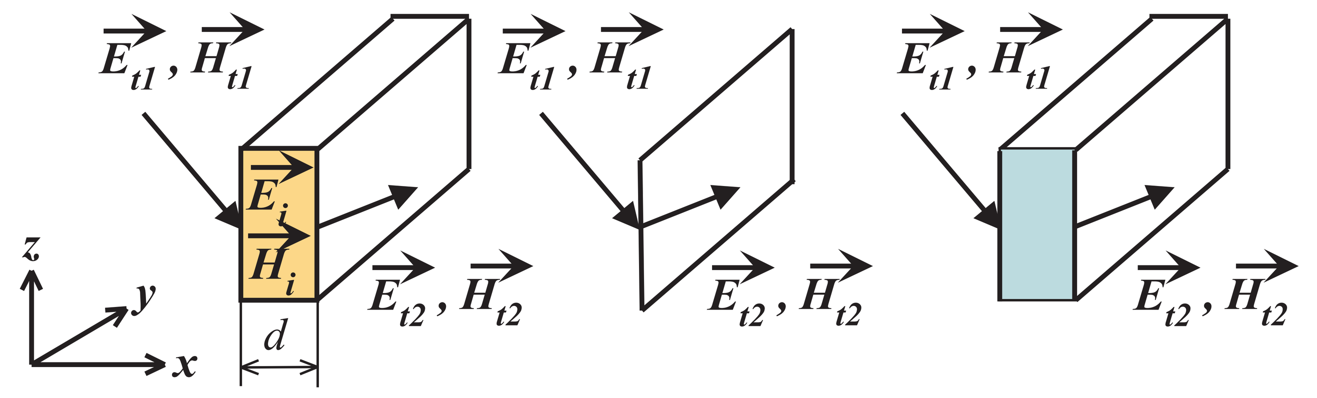
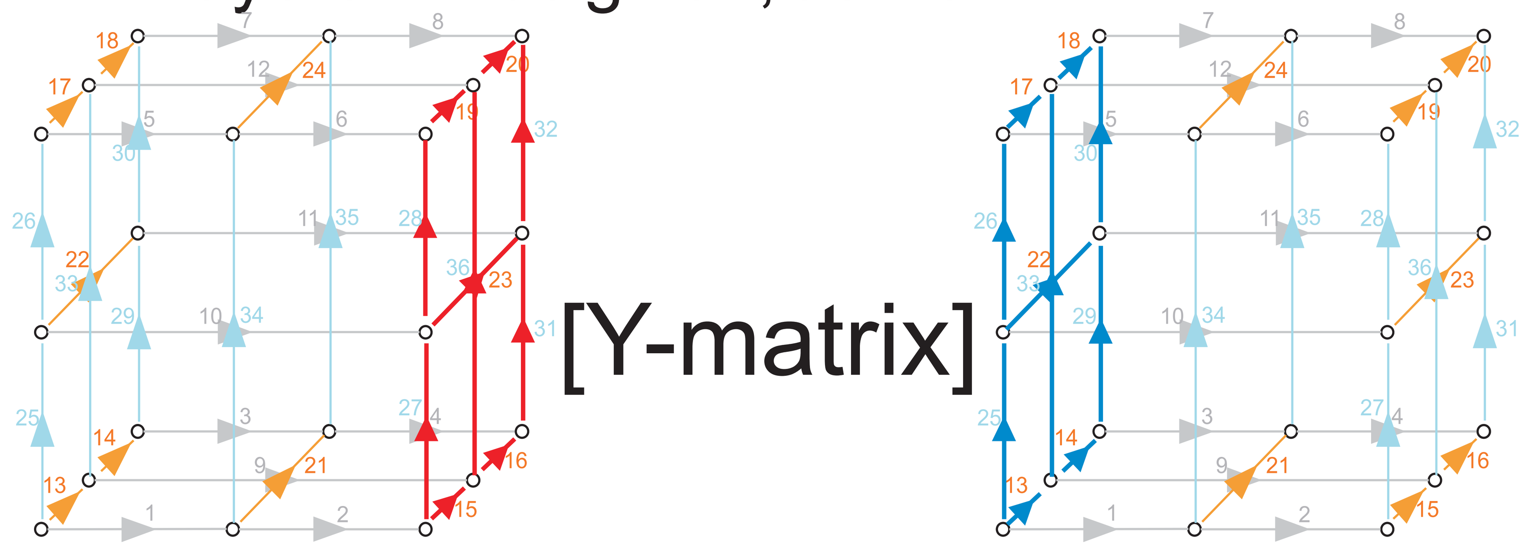
Abstract: The behavior of an electromagnetic wave impinging on a thin layer of arbitrary material can be described with a vector circuit interpretation of a transition matrix model. This paper follows the idea of applying such a matrix to the finite element method (FEM). Especially, several very thin layers composed to an electrically not so thin layer may be treated comfortable. The method will be compared firstly on simple dielectric layer with a known analytic solution. In consequence the example of a thick layer modeled by a number of thin layers will be given, as well.



physical layer thickness= d | sheet model | box model



Network model:

$$\vec{E} = \vec{E}_t + \vec{n}\,E_n,\ \vec{H} = \vec{H}_t + \vec{n}\,H_n,\ \nabla = \nabla_t + \frac{\partial}{\partial n}\vec{n}$$

$$\frac{\partial(\vec{n}\times\vec{E}_t)}{\partial n} = -j\omega\mu\vec{H}_t + \frac{j}{\omega\epsilon}\nabla_t\times(\nabla_t\times\vec{H}_t)$$

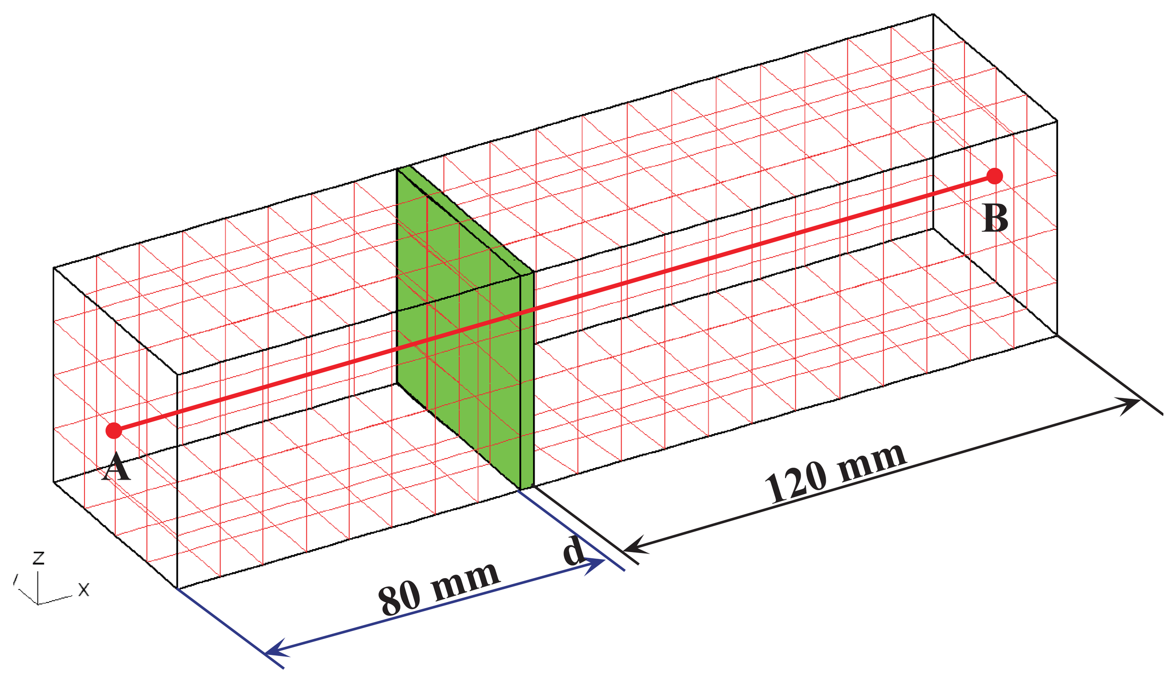
$$\frac{\partial(\vec{n}\times\vec{H}_t)}{\partial n} = j\omega\epsilon\vec{E}_t - \frac{j}{\omega\mu}\nabla_t\times(\nabla_t\times\vec{E}_t)$$

$$\begin{Bmatrix}\vec{E}_{t1}\\ \vec{n}\times\vec{H}_{t1}\end{Bmatrix} = \begin{bmatrix}\bar{a}_{11} & \bar{a}_{12}\\ \bar{a}_{21} & \bar{a}_{22}\end{bmatrix}\cdot\begin{Bmatrix}\vec{E}_{t2}\\ \vec{n}\times\vec{H}_{t2}\end{Bmatrix}$$

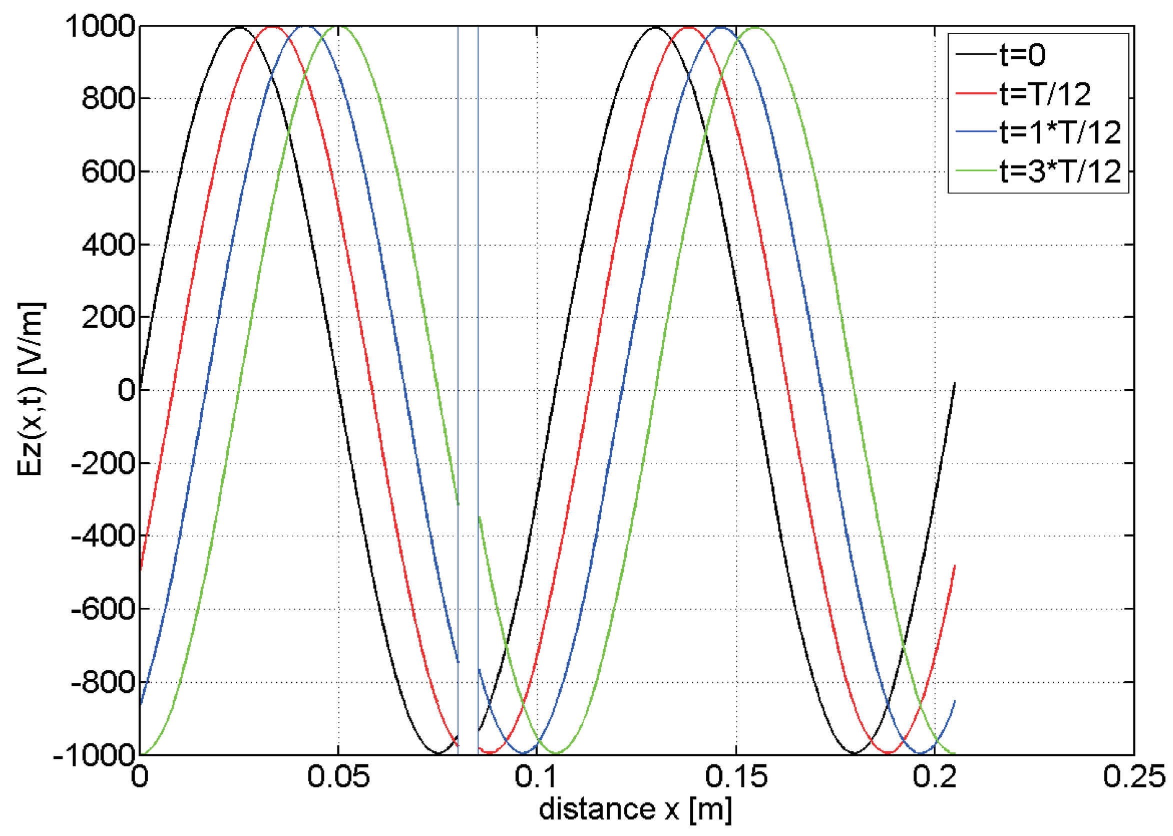
$$a_{11} = \cos(\beta\,d) \qquad a_{12} = j\eta\frac{\beta}{k}\sin(\beta\,d)$$
$$a_{21} = j\frac{k}{\eta\beta}\sin(\beta\,d) \quad a_{22} = \cos(\beta\,d)$$

Galerkin, A,v-formulation:

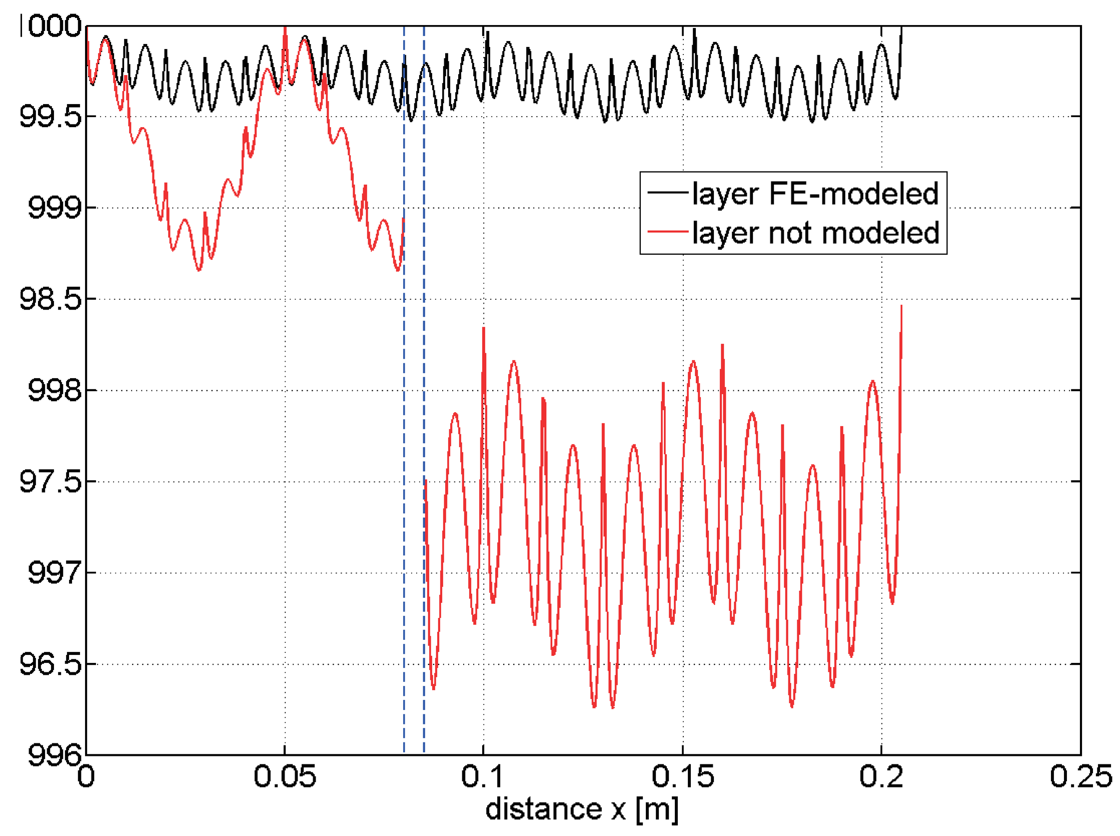
$$-\int_\Omega \nabla\times\vec{N}_i\cdot\frac{1}{\mu}\nabla\times\vec{A}\,d\Omega + \int_{\Gamma_H}\vec{N}_i\cdot(\vec{n}\times(\frac{1}{\mu}\nabla\times\vec{A}))\,d\Gamma$$
$$+\int_\Omega \vec{N}_i\cdot(\sigma + j\omega\epsilon)j\omega(\vec{A}+\nabla v)\,d\Omega = 0$$



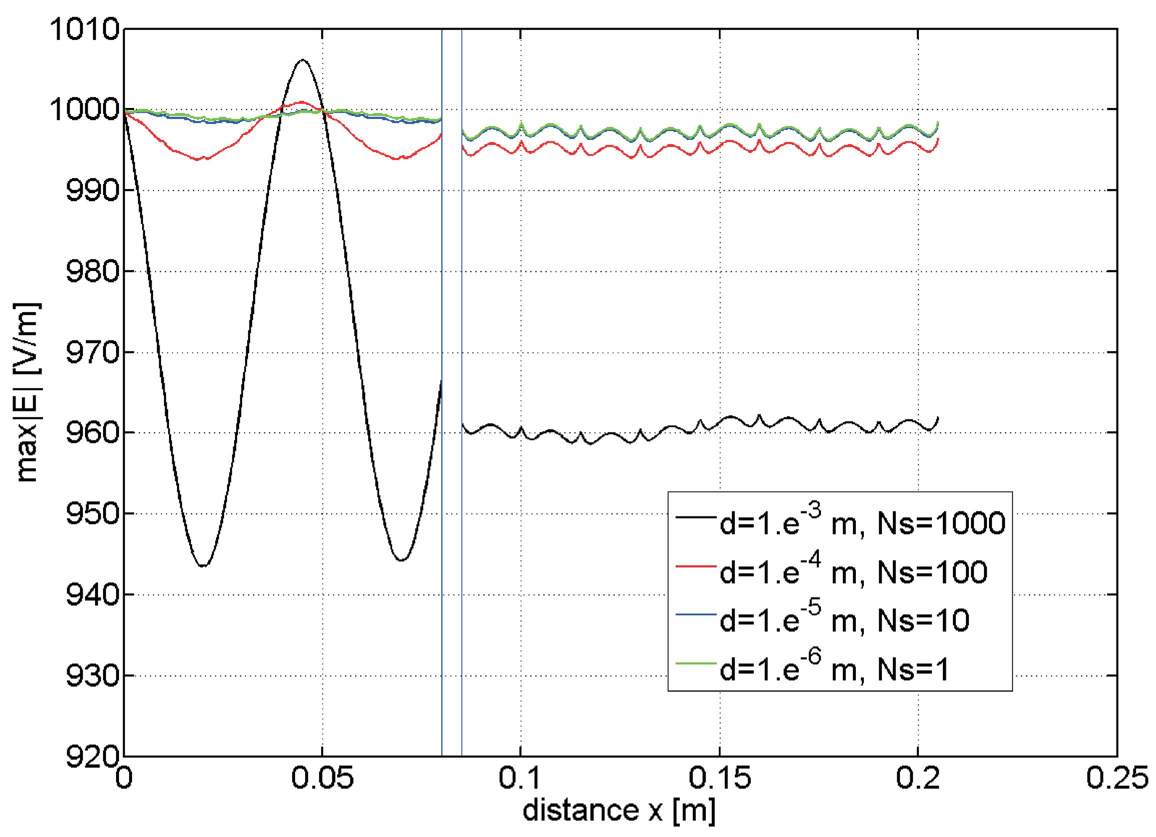
FE-structure



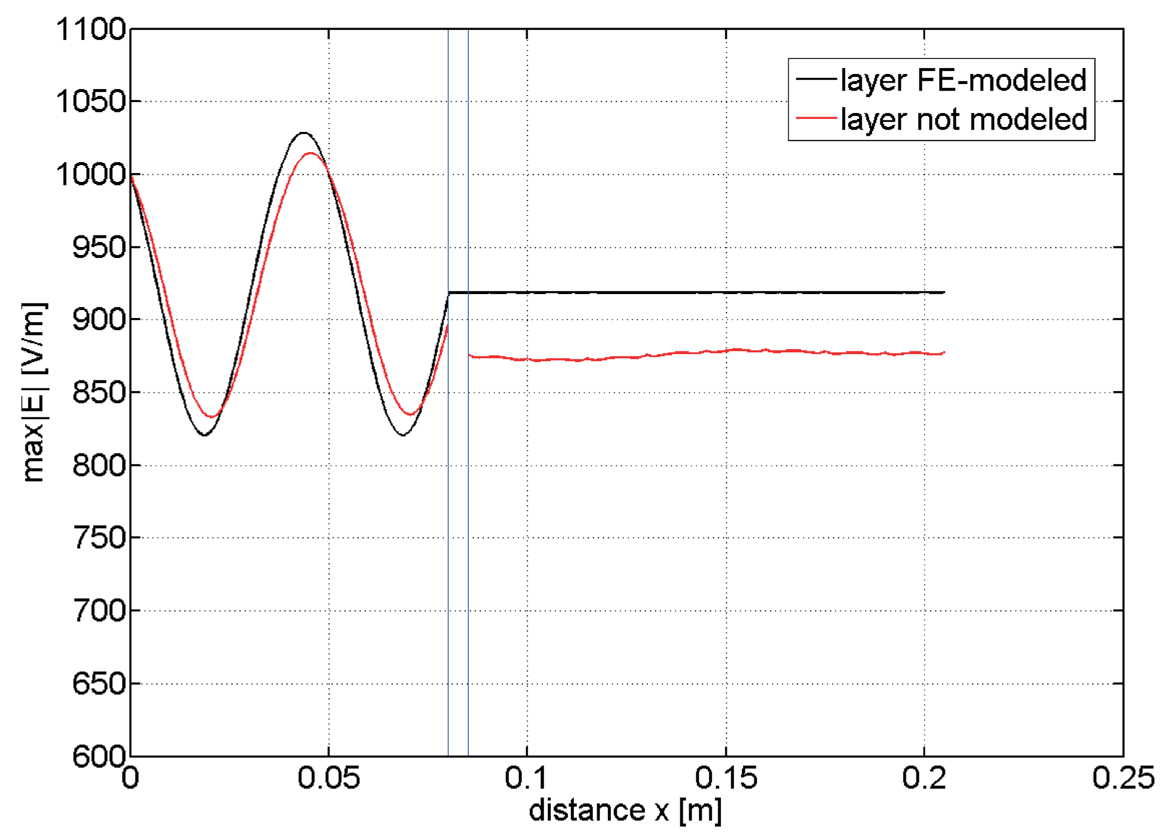
Ez(x,t), matched case, d=10 μm



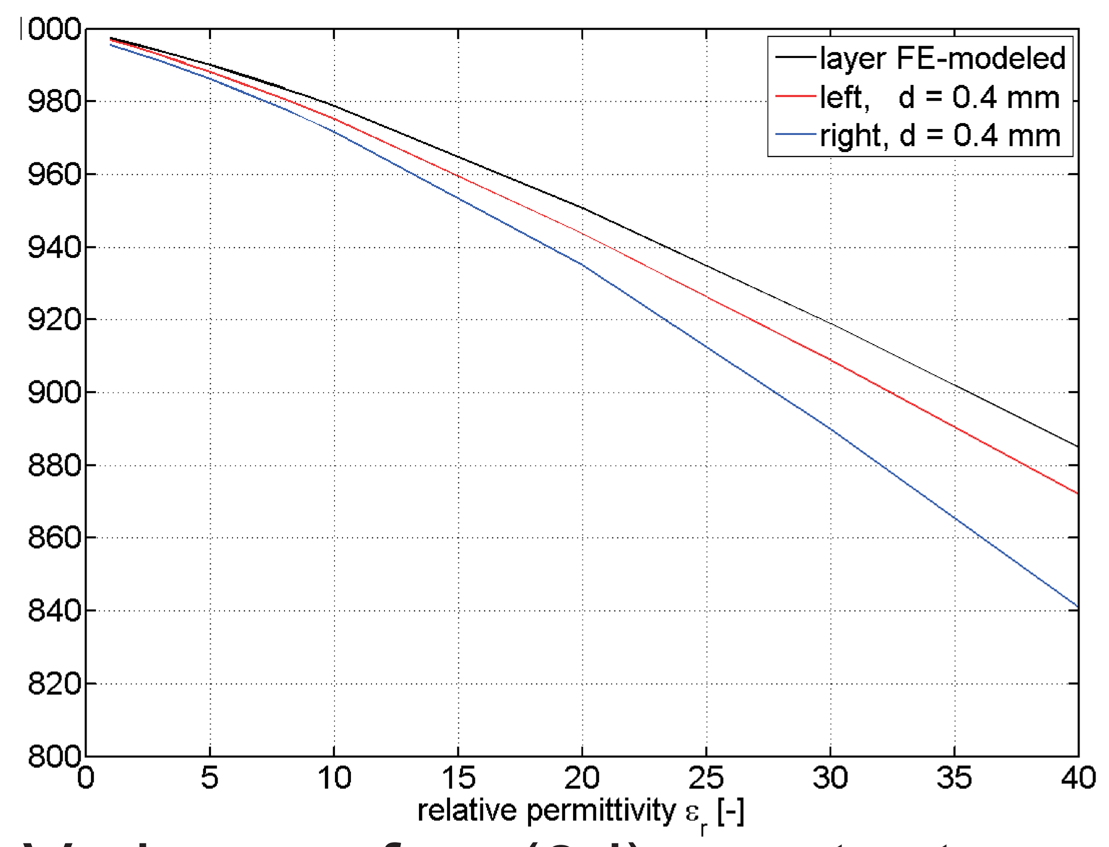
Accuracy of linking, d=10 μm



d variable, (βd) constant



Comparison for ε_r = 10



Variance of ε_r, (βd) constant

Conclusion:

Thin layer network modeled in FEM-implemented

Proper choice of thickness of joint layers influences the quality of linking

Up to now plane wave penetrates the layer perpendicularly, only.