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Effective Dynamic Material Properties of Foam-like Microstructures

S. Alvermann* [1] and **M. Schanz**[1]

[1] Institute of Applied Mechanics, TU Graz, Technikerstr. 4, 8010 Graz, Austria

The effective material parameters of a microstructured material can be found using homogenization procedures based on calculations of a Representative Volume Element (RVE) of the material. In our approach the RVE is calculated in frequency domain and inertia is taken into account, leading to a frequency dependent behavior of the RVE. With the frequency response of the RVE, effective dynamic properties of the material are calculated using an optimization procedure. Due to the frequency dependent material behavior on the microscale a viscoelastic constitutive equation is applied on the macroscale. An example calculation is presented for an auxetic 2-D foam-like microstructure which is modelled as a plane frame structure.

1 Introduction

The determination of macroscopic, effective properties of microstructured materials is referred to as homogenization. There are many different approaches to calculate the effective mechanical properties of a material, a comprehensive overview is given in [1] or [2]. Most publications on homogenization, however, only concentrate on the calculation of statically loaded microstructures where in some cases an analytical solution of the problem is possible.

It is assumed that the structures to be considered are made up of identical unit cells. When assuming periodicity of the microstructure, no effort has to be put on the choice of a Representative Volume Element (RVE) because the smallest repetitive unit of the material is representative. Here, a special interest lies on auxetic materials, i.e., materials that exhibit negative Poissons ratios. The microstructure of such a material usually has re-entrant corners.

The calculation of the unit cell is performed in frequency domain due to a harmonic excitation. Contrary to other approaches, inertia effects on the microscale are taken into account. Thus, in a frequency domain calculation, the response of the unit cell is frequency-dependent which requires the macroscopic constitutive equation also to be frequency-dependent. A frequency domain calculation is preferred against a time domain calculation because periodic boundary conditions can easily be applied. Moreover, the Boundary Element Method is used for the calculation of the RVE because it provides analytical exact results [3].

On the macroscale, a linear viscoelastic constitutive equation is applied. In frequency domain and considering the isotropic case, the constitutive equation

$$\sigma_{ij}(\omega) = \frac{E(\omega)}{(1+\nu(\omega))}\,\epsilon_{ij}(\omega) + \frac{\nu(\omega)E(\omega)}{(1+\nu(\omega))\,(1-2\nu(\omega))}\,\delta_{ij}\,\epsilon_{kk}(\omega) \tag{1}$$

can be applied, where $\sigma_{ij}(\omega)$ and $\epsilon_{ij}(\omega)$ denote the stress and strain and $E(\omega)$ and $\nu(\omega)$ are the complex Youngs modulus and the complex Poissons ratio. For $E(\omega)$ and $\nu(\omega)$, a seven-parameter model with fractional derivatives is used. For the Youngs modulus it is,

$$E(\omega) = \bar{E}\frac{1+q_1(i\omega)^{\alpha_1}+q_2(i\omega)^{\alpha_2}}{1+p_1(i\omega)^{\alpha_1}+p_2(i\omega)^{\alpha_2}} \quad , \tag{2}$$

for $\nu(\omega)$ an analog model is used. $E(\omega)$ can be split up into a real part (storage modulus) and imaginary part (loss modulus). The choice of this model and the number of parameters is somewhat arbitrary, a considerable number of test calculations has shown that this model is sufficient for the present application.

2 Homogenization using an Evolutionary Strategy

The homogenization is formulated as an optimization problem, i.e., 'Find material parameters on the macroscale which describe the micromechanical behavior as good as possible'. The consideration of dynamic effects on the microscale makes it necessary to use an optimization algorithm. As optimization function, the square difference between the microscopic and macroscopic system response is used. The differences are summed up over the considered frequency range. If a harmonic strain is applied on the unit cell, the minimization function is

$$\sum_{\omega=0}^{\omega_{max}} \left[\sigma^{micro}(\omega) - \sigma^{macro}(\omega, E, \nu)\right]^2 \rightarrow 0 \quad . \tag{3}$$

* Corresponding author: e-mail: s.alvermann@tugraz.at, Phone: +43 (0)316 873 7606, Fax: +43 (0)316 873 7641

During the optimization, the stress $\sigma_{ij}^{micro}(\omega)$ must be calculated only once, whereas $\sigma_{ij}^{macro}(\omega)$ is calculated in each iteration step of the optimization procedure via equation (1), using improved values for the sougth parameters. In principle, the procedure can be inverted, i.e., stress can be applied on the cell and the strain response on the macroscopic scale can be optimized.

An Evolutionary Strategy is used to minimize (3). The advantage of this procedure is that local minima do not cause the optimization to fail, for details the reader is referred to [4].

3 Numerical example

The prescribed procedure is applied to the unit cell depicted in Fig. 1 (a). A harmonic strain in one direction is applied to the cell as shown in Fig. 1 (b). The cell consists of beams with a quadratic cross section of $0.1 \cdot \ell \times 0.1 \cdot \ell$. The microstructure material is PMMA. The material parameters are taken from experiments and exhibit a slight damping. The frequency response for an applied load of $\bar{u} = 0.001\ell$ is calculated on a logarithmic scale up to a frequency of $1.5 \cdot 10^7 Hz$. Due to the symmetry of the cell periodicity is fulfilled, e.g., the reaction force of the node on the left hand side corresponds to the applied load on the right hand side. The results of the optimization are depicted in Fig. 2.

Fig. 1 Geometry and loadcase of the unit cell

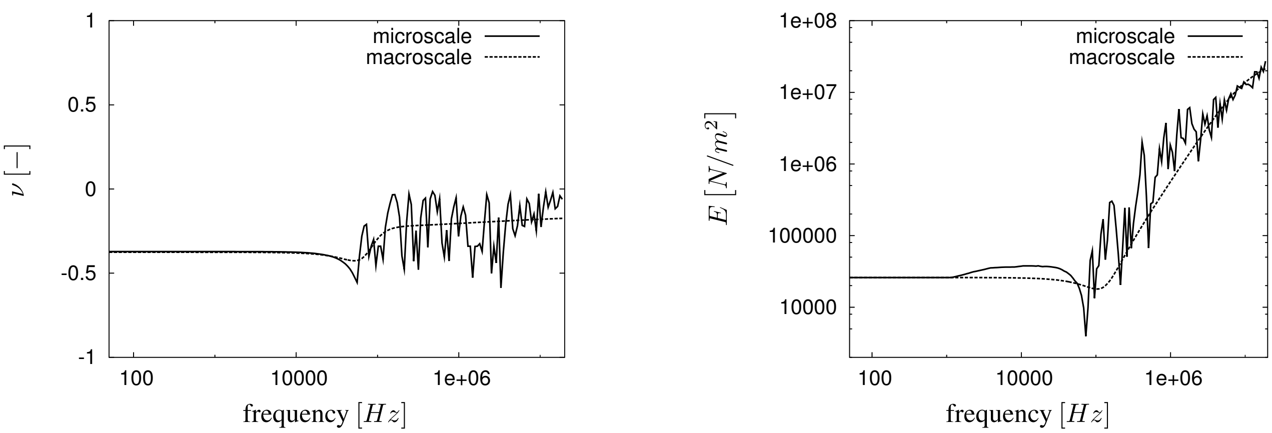

Fig. 2 Homogenization using an Evolutionary Strategy: E and ν on micro- and macroscale plotted versus frequency

E and ν on the microscale (solid line) and on the macroscale (dashed line) are plotted versus the considered frequency range. One can observe that the eigenfrequencies on the microscopic scale can not all be found in detail by the macroscopic constitutive equation. But more important for the homogenization is that the overall tendency is preserved.

For the identified material parameters of $E(\omega)$ and $\nu(\omega)$, no comparison with non-auxetic microstructures was yet made. Therfore, further invesigations with different microstructures have to be carried out. Moreover, the method should be extended to anisotropy, requiring more parameters in the constitutive equation.

References

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