

# TYRE MODELS FOR VEHICLE DYNAMICS ANALYSIS

Edited by

Hans B. Pacejka

*Proceedings of 1st International Colloquium on  
Tyre Models for Vehicle Dynamics Analysis  
held in Delft, The Netherlands  
October 21-22, 1991*

*Supplement to  
Vehicle System Dynamics, Volume 21*



SWETS & ZEITLINGER B.V. AMSTERDAM / LISSE

SWETS & ZEITLINGER INC. BERWYN, PA.

PUBLISHING SERVICE

# Tyre Computation Module DTIRE

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## SUMMARY

The tyre forces computation module DTIRE has been developed in the R&D department of Steyr-Daimler-Puch Fahrzeugtechnik supported by an extensive testing and measurement programme. The major goals of DTIRE were identified as

- tyre forces generator for vehicle dynamics simulation and
- observer model for real time processing,

with regard to the company's main activities in development and production of all-wheel-drive vehicles and system components, respectively. The codes themselves are written in FORTRAN 77 and currently they are linked to the multibody simulation system DADS. However, in principle the tyre processor works autonomously.

The semi-empirical model is based on the *Magic Formula* approximation technique for the description of tyre forces and torques. One necessary extension concerns asymmetrical force-slip characteristics, as they are typical for tyres of direction oriented type. A simplified interpolation approach is taken, in order to perform computations efficiently. Some selected examples of the application of the briefly presented methods are included in the paper.

## 1. INTRODUCTION

Contact forces and torques acting between tyres and the road are primarily responsible for a vehicle's dynamical behaviour, and, in particular for its driving stability. There is an ever increasing interest in the accurate and efficient modelling of tyre force reactions with respect to vehicle dynamics simulation. Due to its engagement in the European PROMETHEUS project, Steyr-Daimler-Puch Fahrzeugtechnik is interested in addressing these important problems. While focussing on safety aspects of all-wheel-drive vehicles, the tyre force computation module DTIRE has been developed, supported by an extensive measurement and testing programme, [3], [4].

The structure of the program system is shown in fig. 1. The codes themselves are written in FORTRAN 77 and currently they are linked to the multibody simulation system DADS [2], but in principle DTIRE works autonomously. The kernel modules serve for the computation of both the tyre contact forces and the current effective tyre radius. Obviously, the accurate description of the effective radius is as important as force modelling itself when dealing with slip controlled driving or braking systems. However, this aspect is beyond the scope of the present paper.

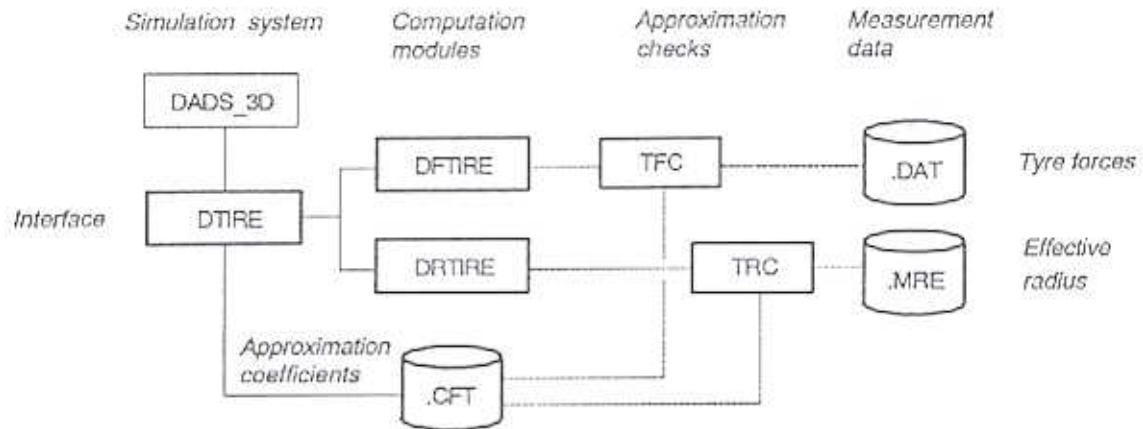


Fig. 1: Structure of the tyre computation system

The kernel modules are called by the simulation program via DTIRE as well as by the auxiliary programs TFC and TRC, which serve to check the quality of any approximation. The coefficient data file Tyre\_Type.CFT keeps the generated set of approximation coefficients for the considered tyre type.

## 2. TYRE FORCE COMPUTATION

### 2.1 TYRE BASIC CHARACTERISTICS

The mathematical description of the basic slip-force/torque characteristics is based on the *Magic Formula* approximation technique [1], [3]. Typically, for this method, the description of the pure longitudinal and lateral forces as well as the aligning torque is performed by the approximation function

$$Y = D \sin[(C \arctan (B\Phi)] + S_v \quad (1a)$$

with the substitution

$$\Phi = (1-E)(X+S_h) + (E/B) \arctan [B(X+S_h)] , \quad (1b)$$

where  $Y$  denotes the current force/torque property depending on the slip variable  $X$ . Furthermore,  $B$ ,  $C$ ,  $D$ ,  $E$  are approximation coefficients depending on the radial load  $F_z$ .  $S_h$ ,  $S_v$  denote the curve shifting parameters. The use of these approximations is a convenient way of mathematically modelling single tyre forces  $F_{lg}$ ,  $F_{lt}$  and pure aligning torque  $T_z$ . Particularly, the approximation coefficients have direct physical

meanings and they are quite easily obtained from measurement curves for any arbitrary tyre type.

In particular, the longitudinal slip/force characteristic is of a typical asymmetrical nature and is caused by the deformation process within the contact zone. Force asymmetries appear in opposite direction of the rolling resistance shift. However, this effect may be intensified by the design of a direction oriented profile and carcass, see fig. 2.

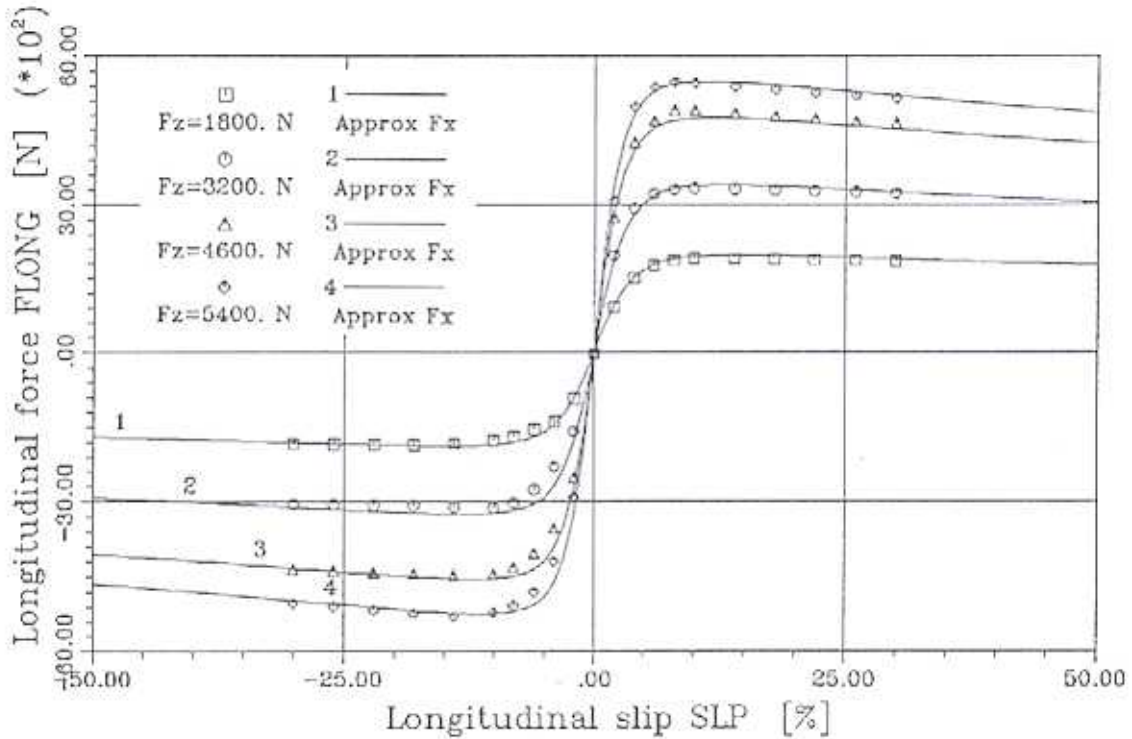


Fig. 2: Pure longitudinal tyre forces (Radial 205/50 R15, rim 6J, p=2 bar, v=60 km/h)

To obtain an asymmetrical curve shape independent on  $F_z$ , the basic approximation function (1) is overlayed for the longitudinal force  $Y := F_{lg}$  by the scaling function

$$S(\text{slp}) = 1 + F \arctan(\text{slp}), \quad (2)$$

such that

$$F_{lg}^*(\text{slp}, F_z) = S(\text{slp}) F_{lg}(\text{slp}, F_z). \quad (3)$$

The additional weighting coefficient  $F$  is easily determined by

$$F = \frac{2}{\pi} \Lambda_{\infty}, \quad (4)$$

where  $\Lambda_{\infty}$  is the scaling factor for large slip values as  $\text{slp} \rightarrow \infty$ . Fig. 2 shows a measurement approximation for a radial tyre 205/50 R15 of direction oriented type using  $F = 0.02$ . The related experimental results were provided by a flat surface bench test.

## 2.2 COMBINED TYRE FORCES

Concerning the combined longitudinal-lateral forces, DTIRE applies a modified method of interpolation between the current basic slip curve values

$F_{x0}$  for pure longitudinal force and

$F_{y0}$  for pure side force,

in order for the method to be highly computationally efficient. Thus, in contrast to [1], only the side slip  $\text{tal} := \tan \alpha$  is transformed into an approximately equivalent slip quantity  $\text{sly}$  with respect to the longitudinal slip property  $\text{slp}$ . This is done in such a manner that the initial derivatives of both basic slip curves  $F_{x0}(\text{slp}, F_z) = F_{lg}^*$  and  $F_{y0}(\text{sly}, F_z)$  are similar for any arbitrary load  $F_z$ . The transformation to the theoretical side slip property  $\text{sly}$  reads

$$\text{sly} = \text{tal} / G(F_z), \quad (5)$$

where the relation

$$G(F_z) = \frac{100 \pi}{180} \frac{B_x C_x D_x(F_z)}{B_y C_y D_y(F_z)} \quad (6)$$

holds with respect to  $\text{tal} \approx \alpha$  for small values of  $\alpha$ . The constant is a result of the change from the unit *degree* slip angle to *percent* side slip.

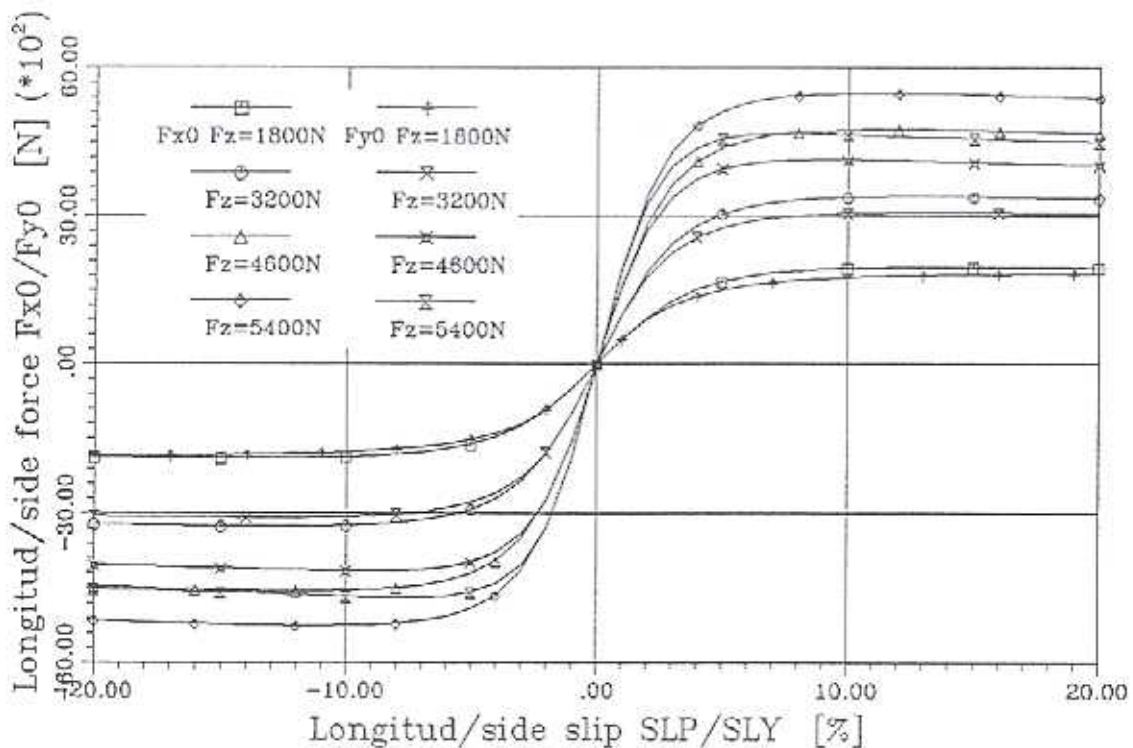


Fig. 3: Basic forces-slip diagram (Radial 205/50 R15, rim 6J,  $p=2$  bar,  $v=60$  km/h)

Thus, the slip vector

$$s = \begin{pmatrix} slp \\ sly \end{pmatrix} \quad \text{for} \quad \begin{array}{l} -\infty < slp < \infty \\ -\infty < sly < \infty \end{array} \quad (7)$$

which still includes the unchanged longitudinal slip component  $slp$ , can be constructed. Now the resulting horizontal force vector  $F$  is assumed to be acting in approximately the opposite direction  $\beta$  of the slip vector  $s$ . Its magnitude  $F$  can be computed by the arbitrarily defined interpolation formula (see fig. 4):

$$F := |F| = 1/2 [F_x' + F_y' + (F_x' - F_y') \cos 2\beta] . \quad (8)$$

Finally, the resulting horizontal contact force  $F$  is given by

$$F = \begin{pmatrix} F \cos \beta \\ F \sin \beta \end{pmatrix} . \quad (9)$$

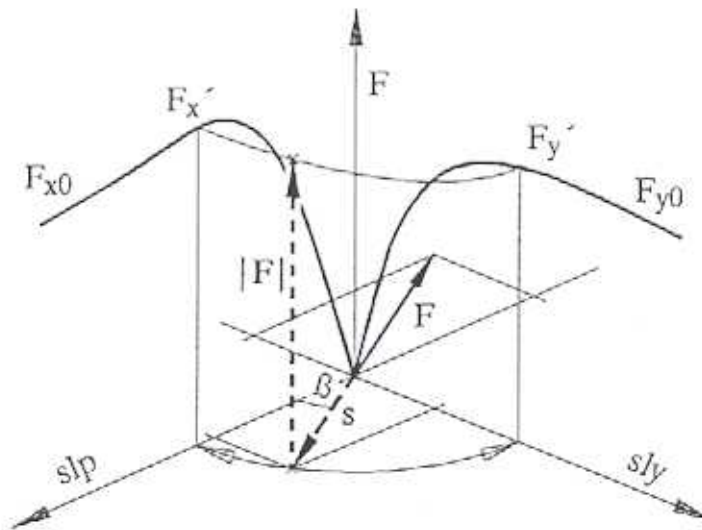


Fig. 4: Combined tyre forces interpolation

Figure 5 shows an example of a horizontal tyre force computation for the referenced tyre type under vertical load  $F_z = 4600$  N.

A comparison between the unsmoothed measurement data and the computation results for slip angle  $\alpha = 5$  deg is shown in figure 6. Obviously, the model provides a sufficient description of the acting steady state tyre forces.

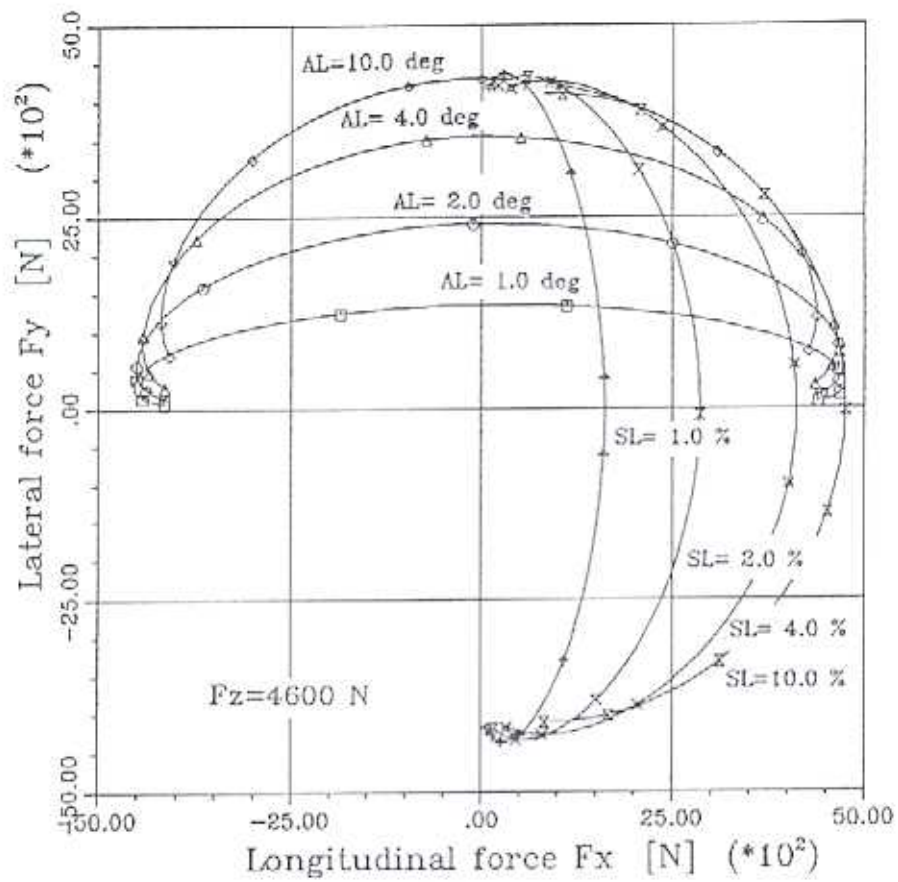


Fig. 5: Horizontal tyre forces (Radial 205/50 R15, rim 6J, p=2 bar, v=60 km/h)

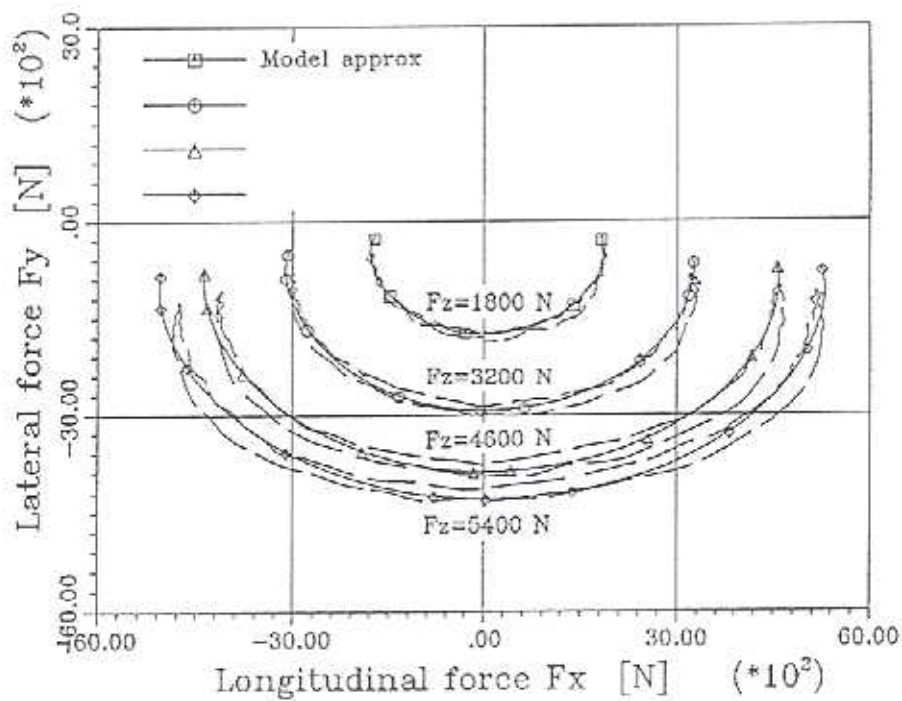


Fig. 6: Computation-measurement comparison

## NOTATIONS

Further notations used:

- slp longitudinal slip  $slp = 100 \frac{\omega r_e - v_x}{v_x} [\%]$
- $r_e$  effective tyre radius
- $v$  translational speed vector of ground point
- $\alpha$  slip angle [deg], clockwise positive
- $\omega$  rotational wheel speed

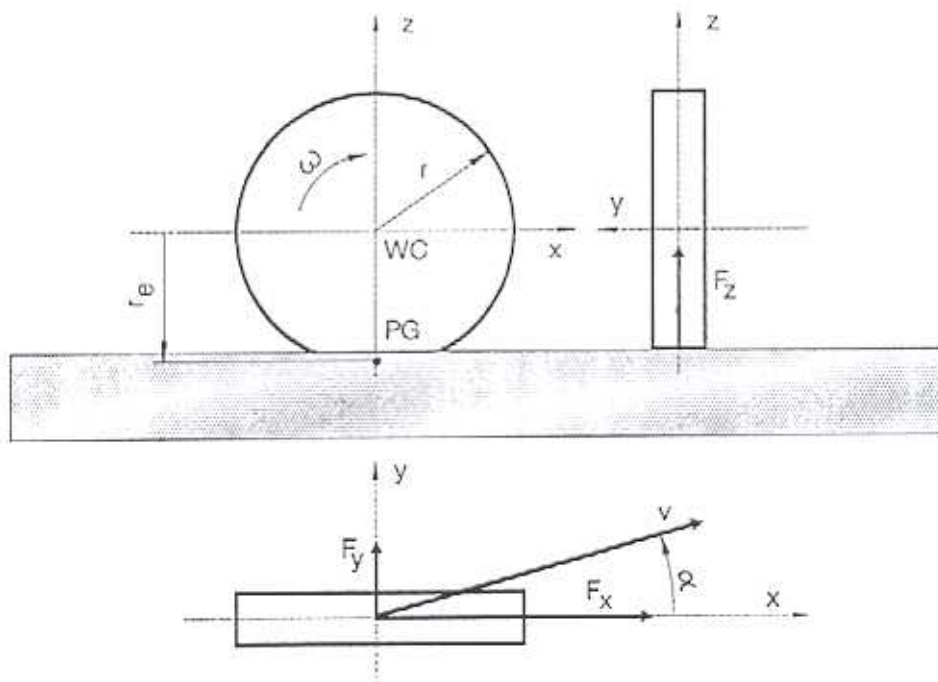


Fig. 7: Definition of positive direction

## ACKNOWLEDGEMENT

The author is indebted to R.Mundl and SEMPERIT REIFEN AG. for their friendly contribution and to J.Holzinger and G.Rieder for their support in testing.



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