INSTITUTE OF THEORETICAL GEODESY AND SATELLITE GEODESY
GRAZ UNIVERSITY OF TECHNOLOGY

Gravity Field Determination using the Acceleration Approach - Considerations on Numerical Differentiation

Norbert Zehentner, Torsten Mayer-Gürr, and Reinhard Mayrhofer

Introduction

Based on Newton's second law of gravitation, the acceleration approach is a method for global gravity field determination using precise orbit information. It directly relates accelerations of a low earth orbiter (LEO) to the gradient of the gravity potential. A core aspect of this approach is the determination of the necessary accelerations. They are produced by numerical differentiation of the orbit positions. These differentiators can be interpreted as finite impulse response filters (FIR).
The presented work investigates various differentiation methods and shows their assets and drawbacks.

Acceleration Approach

The acceleration approach is based on Newton's second law of gravitation. If applied to a satellite's motion, it states that the accelerations $\ddot{r}$ are mainly caused by the gradient of the gravity potential V. The usual representation for the gravity potential is a spherical harmonic series. By exploiting this connection the coefficients can be estimated. *(Austen and Reubelt, 2000)*

$$\ddot{r} = \nabla V$$

$$V(r,\theta,\lambda) = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=0}^{n} P_{nm}(\cos\theta)[c_{nm}\cos m\lambda + s_{nm}\sin m\lambda]$$

One of the main aspects of the whole processing chain is the derivation of the accelerations. This is carried out by means of numerical differentiation.

Numerical Differentiation

Basic formula for polynomial approximation depending on the time difference τ:

$$r(t_0+\tau) = \sum_{n=0}^{N} c_n \tau^n$$

Assuming constant sampling and evaluating the polynomial at the central point leads to the following relation:

$$\ddot{r}(t_0) = 2c_2$$

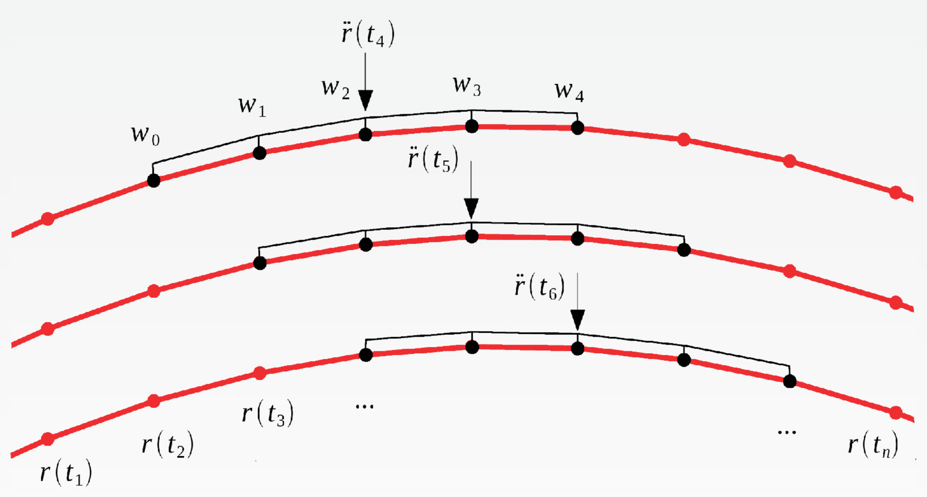


Figure 1: Schematic representation of moving filter function

Additionally, if the points are weighted equally the computation of the second polynomial coefficient, respectively the acceleration, simplifies to a finite impulse response filter. This reduces the complexity and the computational burden.

Power Spectral Density

Different approaches exist to derive accelerations. Three widely used differentiators were investigated:

- Newton-Gregory interpolation
- Taylor-MacLaurin differentiator
- Polynomial approximation

Studies revealed that Newton-Gregory and Taylor-MacLaurin are both a special case of the polynomial approximation. Using a polynomial provides more flexibility in terms of:

- Filter length
- Polynomial degree
- Degree of freedom

An illustrative method for comparing different accelerations, residuals or differentiators is to plot their power spectral density.

Figure 2 shows the power spectral density of accelerations and the filter response of a polynomial with degree 8.

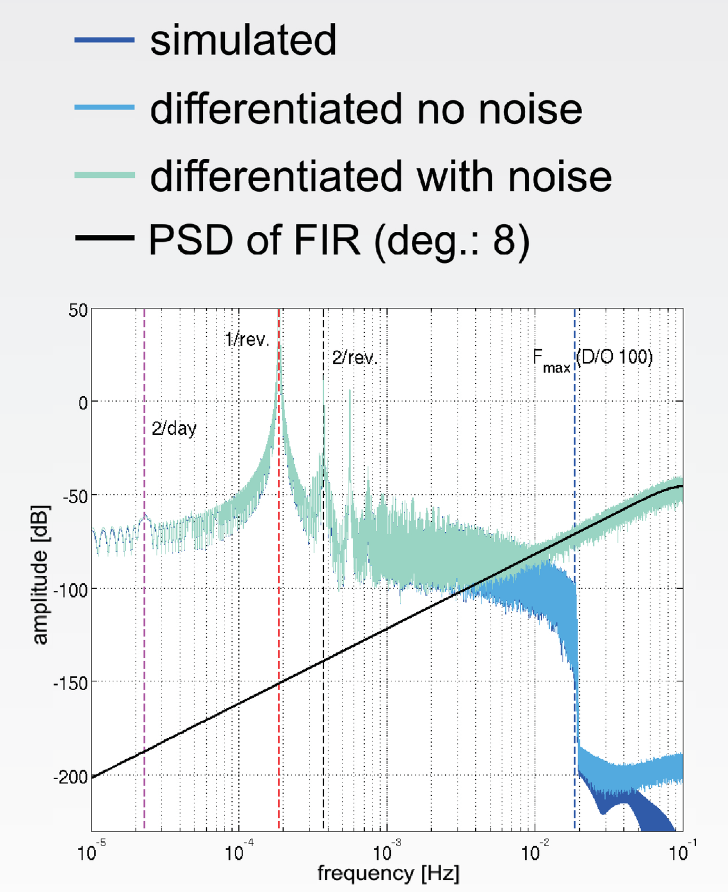


Figure 2: Power spectral density of accelerations

Figure 3 additionally shows the power spectral density of acceleration residuals. A polynomial with degree 8 and different degrees of freedom were applied to simulated positions (2 cm white noise).

- Degree of polynomial: 8
- Used epochs:

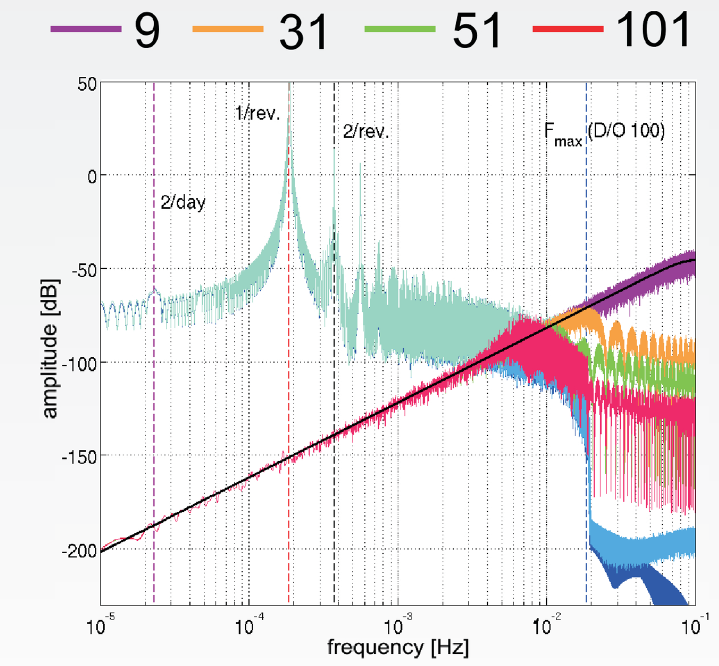


Figure 3: Power spectral density of accelerations and residuals

This comparison clearly shows the effects of overdetermination on the derived accelerations. If it is chosen too high, information is filtered out.
Figure 4 shows the effects on the resulting gravity field estimations based on a simulation scenario:

- GOCE orbit simulated
- 1 month, 5 s sampling
- 2 cm white noise
- Reference model: GOCO02S

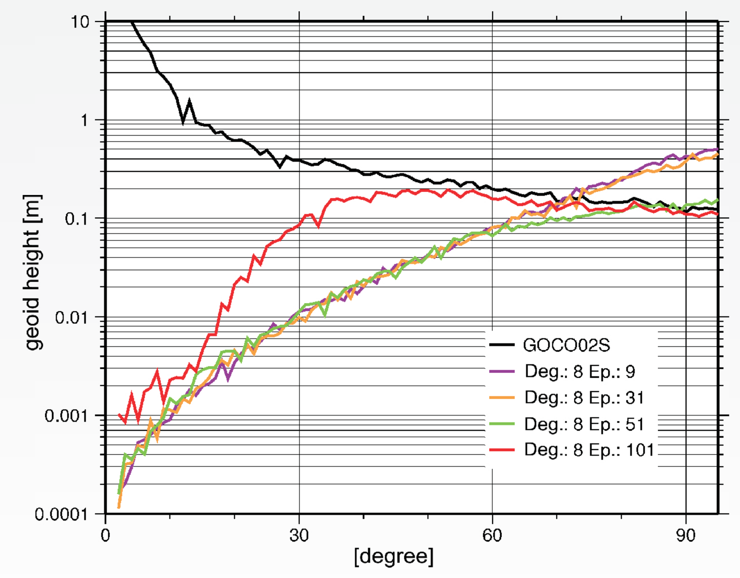


Figure 4: Degree error variances of different closed-loop simulations

Real Data Application: GOCE

Used Data set:

- GOCE precise orbits
- November 2009 - July 2011
- Sampling 5 s

Numerical differentiation:

- Polynomial degree 8
- Used epochs 9

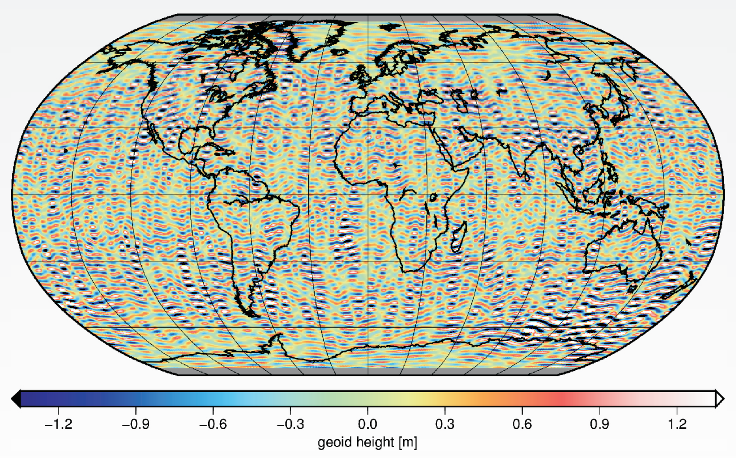


Figure 5: Differences in geoid heights between ITSG solution and GOCO02S

Used background models:

- Tides: direct, earth, pole and ocean tide
- Ocean & Atmosphere: GRACE AOD1B

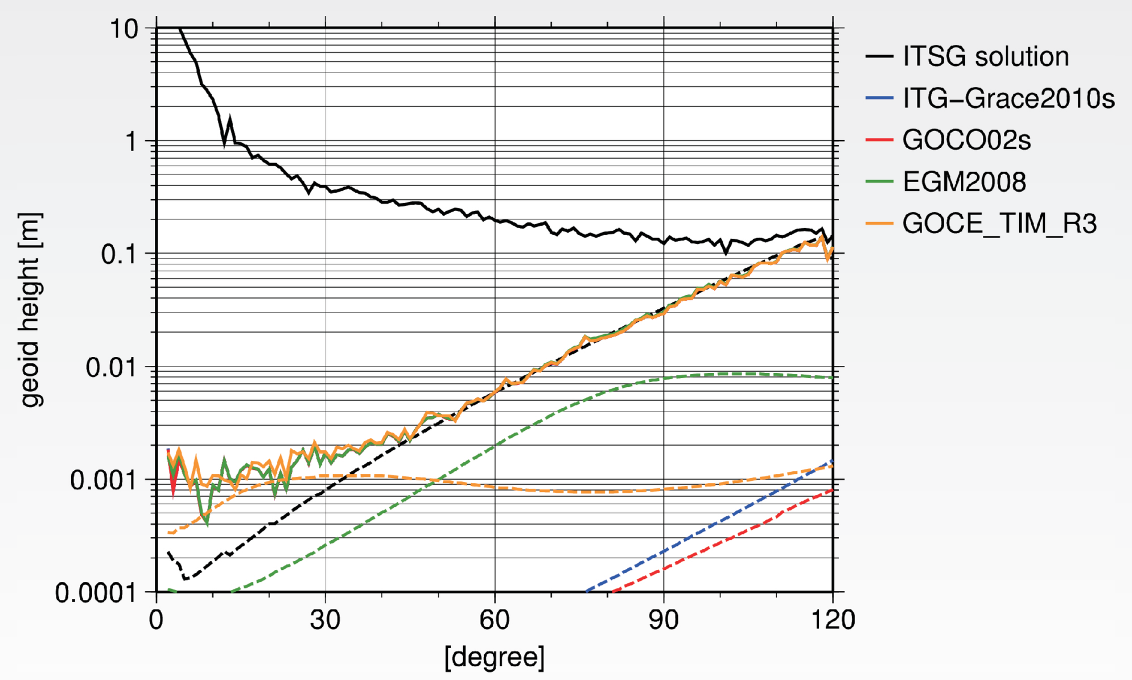
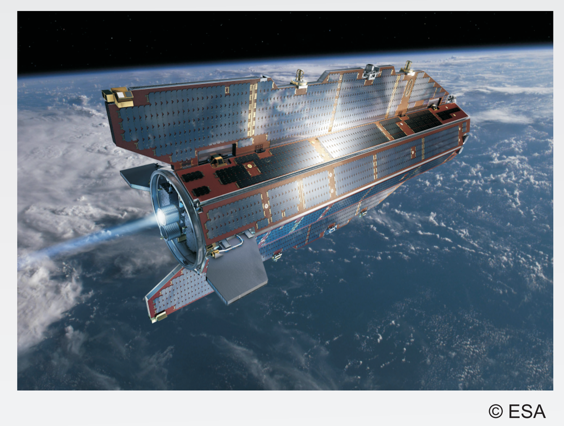


Figure 6: Degree error variances of the ITSG solution compared to ITG-Grace2010s, GOCO02S, EGM2008, and GOCE_TIM_R3. Dashed lines show the corresponding formal degree error variances.

Conclusion



The crucial part of the acceleration approach is the numerical differentiation. It has been shown that the resulting gravity field estimations are not highly influenced by the used method to derive the accelerations. But if an overdetermination is introduced to filter out the influence of noise in high frequencies it can also degrade the solution. This is the case if the filter affects frequencies which still contain a small amount of information.
If a suitable differentiator is used the acceleration approach provides results comparable to other state-of-the-art methods.

Contact

itsg.tugraz.at

Norbert Zehentner
email: zehentner@tugraz.at
phone: +43 316 873 6344

Torsten Mayer-Gürr
email: mayer-guerr@tugraz.at
phone: +43 316 873 6359

Acknowledgement: The presented project Global Gravity Field Modeling from Orbit Data based on the Acceleration Approach (ACAP) was funded by the Austrian Research Promotion Agency (FFG) in the framework of the 7th Austrian Space Applications Program (ASAP).

Reference: **Austen and Reubelt, 2000** : Räumliche Schwerefeldanalyse aus semi-kontinuierlichen Ephemeriden niedrigfliegender GPS-vermessener Satelliten vom Typ CHAMP, GRACE und GOCE, Diplomarbeit, University of Stuttgart