Rotational kinematics of rock blocks with arbitrary geometries

By Markus Pötsch and Wulf Schubert

Introduction

Failure mechanisms of a jointed rock mass in a low or intermediate stress regime are often related to the motion of blocks which are formed by the intersection of discontinuities. Block failure mechanisms are usually investigated with respect to their translational motion. Block theory (1) is a general analysis for these kinds of mechanisms including the investigation of kinematics, failure modes and stability. On the other hand, rotational motion is a fundamental mode which is seldom considered in block analysis. The reasons for this might be:

- The analysis of rotational modes is more complex than the translational analysis.
- Although they are addressed by several authors in the literature, rotational analysis methods are not as common to engineers as probably the translational analyses.
- Rotational modes are not as frequently observed as translational modes.

In the historical development of classification systems for rock slopes rotational failure mechanisms have been successively introduced (2). Typical representatives are the rotational (soil-type) slides, sagging, toppling, or slumping. The consideration of rotational failure mechanisms of blocks and block systems has been recently demanded by Goodman (3) and Goodman & Kieffer (4). They address the failure mechanisms as forward rotation (toppling), backward rotation (slumping), and torsional sliding.

Early works to the rotational analysis of blocks have been done by Londe *et al.* (5) (6) and Wittke (summarised in (7)). A general theory of keyblock rotations has been introduced by Mauldon & Goodman (8) (10) and Mauldon (9). Their theory includes graphical and vector solutions on rotatablity and rotational stability of tetrahedral blocks which are loaded in their centroid. Tonon (11) extended this theory to general loading conditions.

This paper focuses on the kinematical analysis of rock blocks which in general have an arbitrary polyhedral geometry. A method is presented which allows analysing the rotational kinematics of blocks with arbitrary geometries. In order to follow the ideas in previous works ((8) (9) (10) (11)) their terminology has been used. As far it has been necessary for the description of the method, the ideas of the mentioned authors are also shown.

Description of blocks

Rock blocks usually have a complex shape. For kinematical analyses this block shape is approximated by a polyhedron which covers the relevant properties of the original block. There are several kinds of polyhedra.

Kinematik der Rotationsbewegungen von Felsblöcken mit allgemeiner Geometrie

Kurzfassung

Versagensformen von Böschungen in geklüftetem Fels stehen häufig im Zusammenhang mit Bewegungen von Felsblöcken. Die allgemeine Bewegung von Felsblöcken beinhaltet Anteile aus Translation und Rotation. Dieser Beitrag beleuchtet die Kinematik der Rotationsbewegung von Blöcken mit allgemeiner Geometrie. Darauf aufbauend wird eine Methode zur kinematischen Analyse vorgestellt, welche auf früheren Methoden für tetraederförmige Blöcke aufbaut.

Die kinematischen Beziehungen werden durch Vektoroperationen beschrieben, welche zur besseren Illustration in der stereographischen Projektion dargestellt werden. Ein Beispiel an einfachen Blockgeometrien verdeutlicht die Analyseschritte. Weiters wird der Einfluss von ausbeißenden und nicht ausbeißenden Klüften auf den kinematischen Freiheitsgrad diskutiert.

Abstract

A frequently observed failure mechanism of slopes in jointed rock is the motion of blocks. General motion of rock blocks contains contributions from translational and rotational displacements. This paper examines the kinematics of rotational motion of blocks with arbitrary geometries and provides a method for the analysis. The analysis is based on previously proposed methods for tetrahedral blocks.

The kinematical relationships are described by vector operations but for illustration they are represented in stereographic projections. An example on simple block geometries highlights the steps for the analysis and discusses the influence of daylighting and non-daylighting joint planes on the kinematical freedom of blocks.

Geometry

The simplest case of a polyhedron is the tetrahedron. Any polyhedral block can be decomposed into several tetrahedra. It consists of four planes and vertices. Any plane of the tetrahedron can be described by a point in space and a normal vector on the plane. The blockside normal vector of a plane points in direction of the halfspace which forms part of the block. A more general description of a tetrahedron is the convex intersection of four halfspaces defined by the location and blockside normal vectors. This definition can be generalised to polyhedra with *n* faces. A convex polyhedron is formed by the intersection of *n* halfspaces which are defined by the location and blockside normal vectors. Using the denomination of block theory a convex block is described by a code with as many letters as block planes and indicating with $\boldsymbol{U_i}$ (0) and $\boldsymbol{L_i}$ (1) the upper or lower halfspace, respectively.

An even more general block shape is the non-convex polyhedron. It is a polyhedron which contains notches and entrants. It is often referred to as a *complex polyhedron, concave polyhedron*, or *united polyhedron*. As the latter denomination indicates, it can be described by the combination of convex polyhedra. However, the intersection of blockside halfspaces is not a sufficient description for non-convex polyhedra. For an unequivocal description of non-convex polyhedra the vertices of the block, the limits of the planes of the block, and the halfspaces of the planes are required. For the present purpose an advantageous description of arbitrary block shapes is the definition of the block corners and the corresponding blockside normal vectors of the planes forming the corner.

Joints and free surfaces

The geometry of a block gives information of the spatial relationships within the block. For kinematical analyses the support conditions and the free space of the block have to be defined. In the case of a block which is bounded by the rock mass, the support conditions are related to the joint system. A joint represents a contact “block to rock mass”. On the other hand, a free surface represents a contact “block to free space”. Joints constrain block motion while free surfaces facilitate block motion. For the definition of the support conditions information has to be assigned to the block planes whether they are joints or free faces.

Kinematics

General motion of a rigid body

Kinematics basically deals with the location and orientation of systems in space and their variation with time. It is also called the geometry of spatial relationships (12). Motion of rigid bodies can be distinguished into translation and rotation. A translational motion is described by a single vector while a rotational motion is always related to a rotation axis. It is defined by its location and orientation. Both properties of the rotation axis can be either fixed or variable with time.

The general case of motion of a point of a rigid body can be expressed by equations [1] and [2]. They contain the translational and rotational parts. Relative velocities and accelerations between points of the rigid body disappear.

$$\overrightarrow{v_P} = \overrightarrow{v_0} + \vec{\omega} \times \overrightarrow{OP} \qquad [1]$$

$$\overrightarrow{a_P} = \overrightarrow{a_0} + \dot{\vec{\omega}} \times \overrightarrow{OP} + \vec{\omega} \times \left(\vec{\omega} \times \overrightarrow{OP}\right) \qquad [2]$$

$\boldsymbol{v_P}$ ... velocity of block point
$\boldsymbol{a_P}$ … acceleration of block point
$\boldsymbol{v_O}$ … velocity of reference point
$\boldsymbol{a_O}$ … acceleration of reference point
$\boldsymbol{OP}$ .. vector pointing from reference point to block point
$\boldsymbol{\omega}$ ... angular velocity
$\dot{\boldsymbol{\omega}}$ ... angular acceleration

For the determination of the rotatability the incipient motion of the block is considered. Following assumptions are taken into account (11):

- The block is initially at rest, i.e. it has no initial velocity and angular velocity. $\boldsymbol{v_P}$ and $\boldsymbol{\omega}$ vanish.
- The rotation axis is fixed; therefore, it has no initial acceleration. Therefore, $\boldsymbol{a_O}$ vanishes. This applies for situations where the rotation axis passes through a corner or an edge at the free surface. For slumping modes this assumption is not really true, since the rotation axis experiences an acceleration in order to comply with the kinematical constraints of slumping.

If these assumptions hold, the expressions for the initial motion of a block simplify to equation [3]:

$$\overrightarrow{a_P} = \dot{\vec{\omega}} \times \overline{OP} \qquad [3]$$

The assessment of the kinematical freedom of a block focuses on the displacements at the beginning of block motion (initial displacements). A block has a kinematical freedom if it does not collide with the rock mass during its initial displacement. As long as the above assumptions apply, the initial displacement vectors (after an infinitesimal time increment) are parallel to the initial acceleration of the block while the rotation axis is parallel to the angular acceleration vector. This is also called a pure rotation.

Vector formulation

For a block to be kinematically free it must not interpenetrate the rock mass throughout its motion. A block is called rotatable if there is a bundle of rotation axes which allow for kinematically admissible block displacements, in other words, no interpenetration of the block and rock mass takes place. The direction of the initial displacements of a block is unequivocally defined by the displacements of the block corners. The general displacement of a corner can be described by equation [4]:

$$\Delta\vec{x} = \vec{r} \times \left(-\overrightarrow{C_j R}\right) \qquad [4]$$

r is the vector of the rotation axis, $\boldsymbol{C_jR}$ is a vector pointing from the block corner to an arbitrary point on the rotation axis, $\boldsymbol{\Delta x}$ is the initial displacement vector of a corner. The rotation axis may have a remote position to the block but has to be fixed. Mauldon (9) described four possible locations of the rotation axis.

- The rotation axis penetrates the block at minimum one joint. In this case block rotation is not possible unless the block is formed by only one joint (7).
- The rotation axis coincides with a block edge at the transition between joints and free faces. The rotation is referred to as *edge rotation*.
- The rotation axis passes through a block corner at the transition between joints and free faces. This rotation is referred to as *corner rotation*.
- The rotation axis is at a remote location to the block. This rotation is referred to as *remote axis rotation*.

In this paper we focus on corner and edge rotation where the rotation axis is fixed to the block. The vectors $\boldsymbol{C_jR}$ can be determined from the block geometry, in the case of a tetrahedron even from the orientations of the block planes only. The derived principles would also apply to the analysis of remote rotation axes; however, finding suitable vectors $\boldsymbol{C_jR}$ is cumbersome and the rotation axis cannot generally be assumed to be fixed.

A corner displacement is kinematically admissible if it points in the same direction as the blockside normal vectors of the planes forming the corner (Figure 1). In other words, the corner displacement has to plot inside the joint pyramid of the corner. A corner can be formed by three joints, two joints and one free surface, one joint and two free surfaces, as well as three free surfaces. Displacements have to be judged only for corners containing at least one joint. This is indicated by the index ***j*** in equation [4]. The initial displacements have to comply with the conditions in equation [5]:

$$\Delta\vec{x} \bullet \vec{n}_{i,c} = \vec{r} \times \left(-\overrightarrow{C_j R}\right) \bullet \vec{n}_{i,c} \geq 0 \qquad [5]$$

This expression can be mathematically rearranged and leads to the following necessary and sufficient condition on rotatability of blocks for fixed rotation axes (Equation [6]):

$$\left(\vec{n}_{i,c} \times \overrightarrow{C_j R}\right) \bullet \vec{r} \geq 0 \qquad [6]$$

$\boldsymbol{C_j}$ is a vector pointing to a block corner whose intersection planes contain at least one joint. ***R*** is a vector pointing to an arbitrary point on the rotation axis; however, for our purposes it is assumed that it points to the static rotation point on the block. $\boldsymbol{C_jR}$ is the vector pointing from a block corner to the static point at the block containing the rotation axis. $\boldsymbol{n_{i,c}}$ is a blockside normal vector of a joint plane of corner $\boldsymbol{C_j}$. ***r*** is the vector of the rotation axis. Equation [6] has to comply simultaneously for all corners formed by one, two or three joint planes and their corresponding blockside normal vectors. It has to be investigated for all corners at the transition between rock mass and free faces. In the case of a tetrahedron equation [6] simplifies and complies with the equations [13a,b,c] in (10).

The aim of the kinematical analysis is to find possible vectors ***r*** for rotation axes which comply with equation [6]. The sum of all possible vectors is called the *rotation space* (8). It can be divided into rotation spaces for the rotation corners. A block is therefore called *corner rotatable* if the rotation space is not empty. In this paper the rotation space corresponds to right-handed rotations. If there is a possible rotation axis whose vector complies with the rotation spaces of two corners and the intersection of both corners corresponds to a block edge, then rotation about this edge is possible. In this case the block is called to be *edge rotatable*. In consequence, edge rotation is a corner rotation about two corners simultaneously.

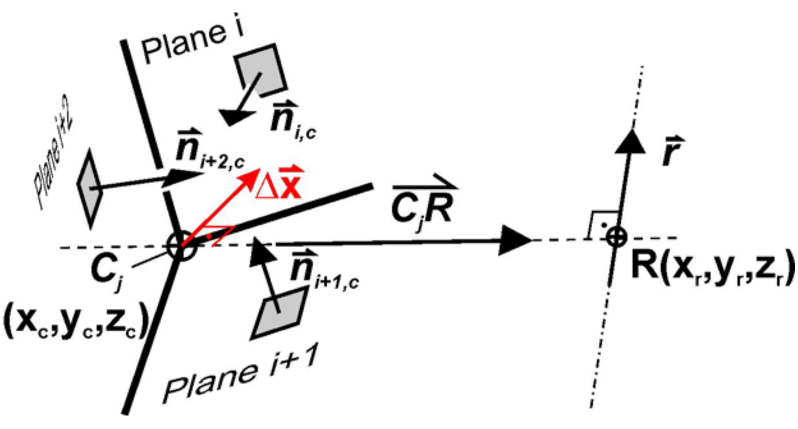

Fig. 1 Corner displacement *Δx* due to rotational motion about rotation axis *r*. For *Δx* to be kinematically admissible it has to point into the halfspaces defined by the joint planes of the corner.

Bild 1 Eckverschiebung Δx aufgrund einer Rotationsbewegung um die Rotationsachse r. Eine kinematisch zulässige Verschiebung Δx zeigt in die Halbräume der Kluftflächen der Ecke.

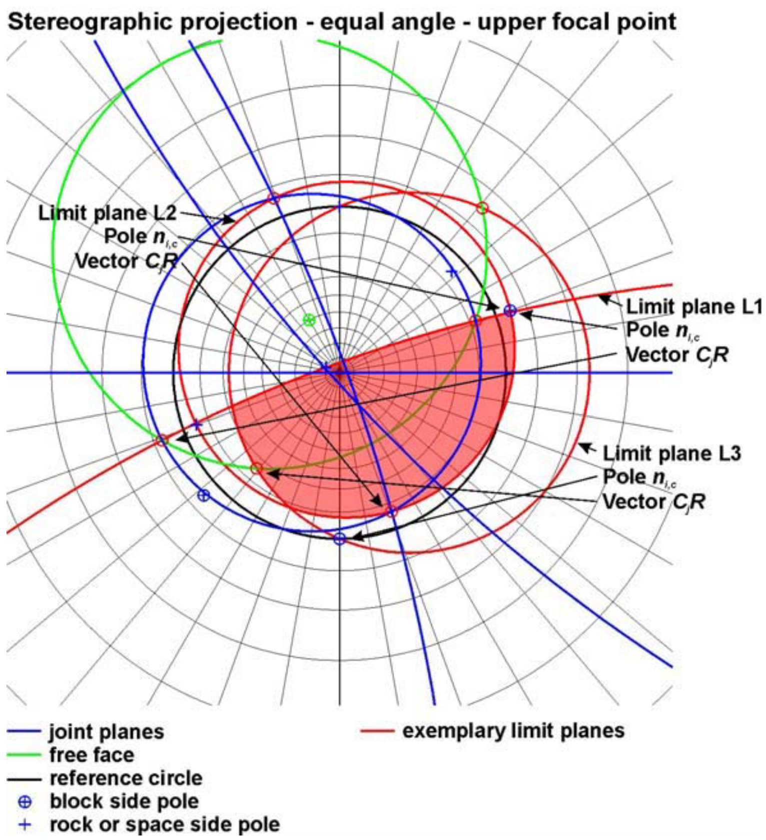

Fig. 2 Great circles of limit planes include the corresponding blockside pole $n_{i,c}$ and the vector C_jR. The latter can be an intersection a joints and free face (L1), or two joints (L2), as well as a vector between two corners which has to be determined separately (L3).

Bild 2 Die Großkreise der Grenzflächen beinhalten den zugehörigen blockseitigen Pol $n_{i,c}$ und den Vektor C_jR. Letzterer kann aus einem Verschnitt von einer Kluftfläche und einer freien Oberfläche (L1) oder zweier Kluftflächen (L2) bestimmt werden. Für komplexere Blockgeometrien muss der Vektor C_jR zusätzlich aus zwei Ecken bestimmt werden (L3).

Graphical representation

A closer examination of equation [6] reveals that the cross product of the vectors $\boldsymbol{n_{i,c}} \times \boldsymbol{C_jR}$ is a vector with at least one component pointing in the same direction as the rotation axis. Conversely, the directions of possible vectors complying with equation [6] can be represented by a plane and a corresponding halfspace. This plane is subsequently referred to as the *limit plane*. As a consequence the rotation space of a corner is the intersection of all halfspaces of relevant limit planes. For the determination of block rotatability the emptiness of the rotation space in terms of intersections of limit planes has to be computed. This is illustrated using stereographic projections of planes and edges (Figure 2). The limit plane is represented by a great circle through the blockside pole of a corner's joint plane and the vector pointing from the block corner to the static rotation corner. While the blockside pole can be directly determined from orientation data, the vector $\boldsymbol{C_jR}$ usually depends on the block geometry and has to be determined separately. Nevertheless, for tetrahedra it is the intersection of two corresponding block planes.

The exemplary construction of limit planes is shown in Figure 2. The great circle of limit plane L1 includes the pole and a block edge (intersection of a joint and free face). The great circle of limit plane L2 passes through the pole and a block edge (intersection of two joints). The great circle of limit plane L3 goes through another pole and a vector whose direction has to be determined from the corresponding block corner coordinates. Since the vector lies on the free face' great circle, it must be included in the free face. The appropriate halfspaces can be determined by turning the pole $\boldsymbol{n_{i,c}}$ along the smaller angle to the vector $\boldsymbol{C_jR}$. According to the right-hand rule, the resulting vector indicates the halfspace. The rotation space under the constraint of these limit planes is shaded in red.

The number of relevant limit planes varies according to the complexity of the block geometry. Corners formed by three joint planes ly inside the rock mass. For each rotation point three limit planes have to be considered. Corners with one or two joint planes lie on the free face. For each rotation point one or two limit planes have to be considered, respectively. Note that there are no limit planes for the current (static) rotation point. A tetrahedron with three joint planes and one free face has 21 limit planes to be considered.

Illustration

The application of the kinematical analysis for rock blocks is shown for an example based on the case study in (11). In this illustration the original block is considered as a maximum block. A daylighting and a non-daylighting joint additionally intersect the original block. The resulting blocks are displayed in Figure 3, the data on the block corners and plane orientations are reported in Table 1 and 2, respectively.

The analysis of the kinematics of the blocks includes removability, as well as corner and edge rotatability. It is studied for three variations:

- Rotation space for the original block omitting joint planes 4 and 5. Block code $U_1U_2U_3L_f$.
- Rotation space for the blocks formed by the intersection of the original block with joint plane 4 (daylighting joint plane). Two blocks

Table 1 Data on block corner coordinates for the blocks shown in Figure 3.

Tabelle 1 Koordinaten der Blockecken für die Blöcke in Bild 3.

Corners	X [m]	Y [m]	Z [m]
C1	8.332	-11.869	0.934
C2	0.755	5.000	16.371
C3	52.612	5.000	-5.386
C4	2.000	5.000	3.000
C5	29.687	-3.734	-2.114
C6	22.954	5.000	7.057
C7	21.999	5.000	-0.314
C8	3.931	-0.145	2.370
C9	1.207	5.000	11.522
C10	12.073	5.000	1.331

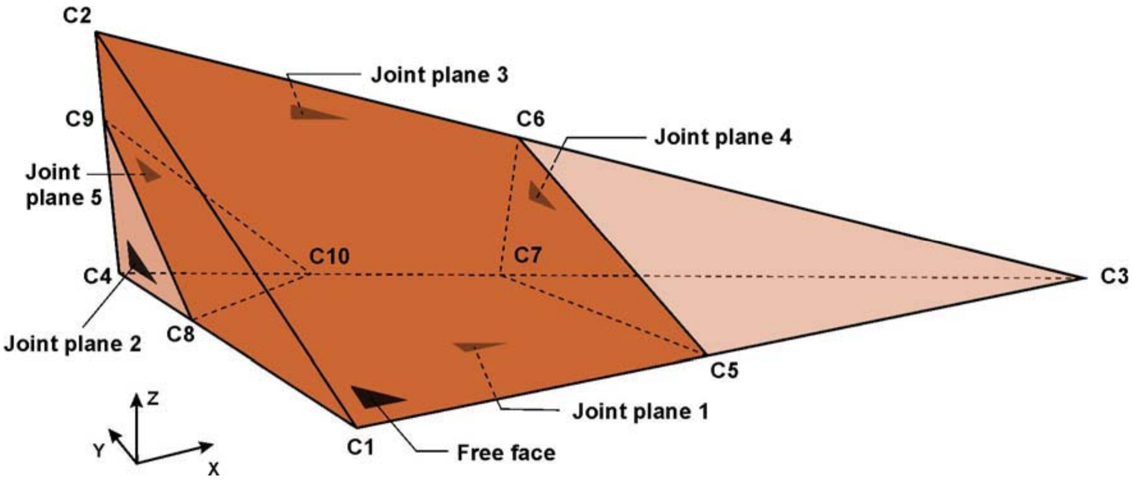

Fig. 3 Overview of block geometries subjected to kinematical analyses. (1) Entire tetrahedron, (2) two polyhedra cut by joint plane 4, and (3) one polyhedron cut by joint planes 4 and 5.

Bild 3 Übersicht über die kinematisch untersuchten Blockgeometrien. (1) Gesamter Tetraeder, (2) zwei Polyeder, die von Kluftfläche 4 geschnitten werden, und (3) ein Polyeder, der von den Kluftflächen 4 und 5 geschnitten wird.

Table 2 Data on the block plane orientations and halfspaces for the blocks shown in Figure 3.

Tabelle 2 Orientierungen und Halbräume der Ebenen für die Blöcke in Bild 3.

Planes	Dip direction [°]	Dip angle [°]	Halfspace
Joint plane 1	110.0	10.0	Upper
Joint plane 2	70.0	85.0	Upper
Joint plane 3	180.0	90.0	Upper
Joint plane 4	227.8	84.5	Upper/Lower
Joint plane 5	143.8	57.8	Upper/Lower
Free surface	150.0	40.0	Lower

are formed; block codes $U_1U_2U_3U_4L_f$ and $U_1U_2U_3L_4L_f$.

- Rotation space for the blocks formed by the intersection of the original block with joint planes 4 (daylighting) and 5 (non-daylighting). Three blocks are formed; block codes $U_1U_2U_3U_4U_5L_f$, $U_1U_2U_3L_4U_5L_f$, and $U_1U_2U_3U_4L_5L_f$. Only the first one is subject to the kinematical analysis. The second one has already been analysed above while the third one is a joint block without any free face.

Original block – Tetrahedron

The corners ***C1***, ***C2***, ***C3***, and ***C4*** form the block $U_1U_2U_3L_f$. Potential rotation corners are those at the free face (***C1***, ***C2***, ***C3***) and potential rotation edges are those between (***C1-C2***, ***C1-C3***, ***C2-C3***). The rotation space is formed by the intersection of the halfspaces of the limit planes. For each corner rotation the rotation space is bounded by seven limit planes; three limit planes for the rock mass point ***C4*** and two for each moved corner at the free face. In total 21 limit planes have to be intersected for the determination of the block's rotation space. In the case of the tetrahedron the vectors pointing from the corners to the rotation axis are defined by the intersection of the block planes (JP edges as well as intersections of joints and free face). These vectors can be directly determined from the stereographic projection.

Figure 4 shows the stereographic projection of joints and free face including poles, as well as the block's joint pyramid and rotation space. The rotation space refers to a right-handed rotation. Since the block's joint pyramid JP000 plots entirely into the space pyramid (region outside the great circle of free face U_f), the block is removable. This is a necessary condition for rotatability of three-joint blocks. The block is corner-rotatable about rotation axes passing through any of the corners at the free face and plotting inside the corresponding rotation space. The block is also edge-rotatable about the edge between ***C1-C3***. The necessary and sufficient condition for this edge rotation is that ***E13*** plots inside the rotation spaces for ***C1*** and ***C3***. No other block edge at the free face simultaneously plots inside two rotation spaces.

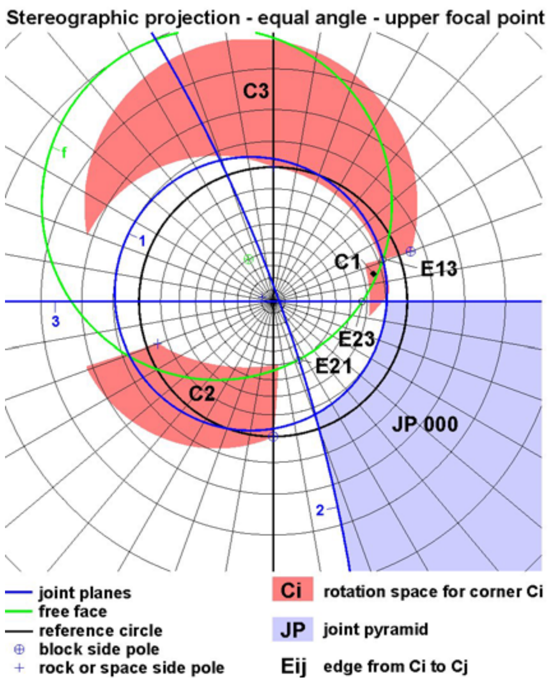

Fig. 4 Stereographic projection of joint planes, free face, joint pyramid and rotation space for the tetrahedron $U_1U_2U_3L_f$.

Bild 4 Stereographische Projektion der Kluftflächen, freien Oberfläche, Kluftpyramide und des Rotationsraums für das Tetraeder $U_1U_2U_3L_f$.

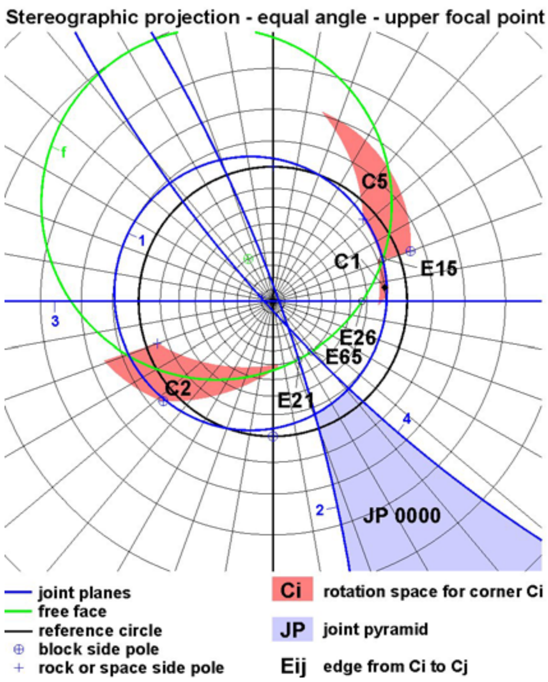

Fig. 5 Stereographic projection of joint planes, free face, joint pyramid and rotation space for the polyhedron $U_1U_2U_3U_4L_f$.

Bild 5 Stereographische Projektion der Kluftflächen, freien Oberfläche, Kluftpyramide und des Rotationsraums für das Polyeder $U_1U_2U_3U_4L_f$.

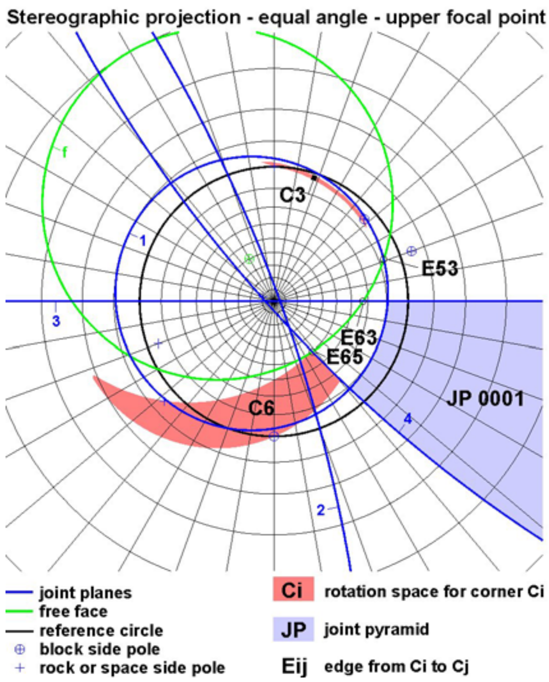

Fig. 6 Stereographic projection of joint planes, free face, joint pyramid and rotation space for the tetrahedron $U_1U_2U_3L_4L_f$.

Bild 6 Stereographische Projektion der Kluftflächen, freien Oberfläche, Kluftpyramide und des Rotationsraums für das Tetraeder $U_1U_2U_3L_4L_f$.

Intersection with a daylighting joint plane

If joint plane 4 intersects the original block, the blocks $U_1U_2U_3U_4L_f$ and $U_1U_2U_3L_4L_f$ are formed. The first block consists of the corners ***C1***, ***C2***, ***C5*** and ***C6*** at the free face, as well as ***C4*** and ***C7*** inside the rock mass. It is a convex polyhedron. The second block consists of the corners ***C3***, ***C5*** and ***C6*** at the free face, as well as ***C7*** inside the rock mass. It is a tetrahedron.

Convex polyhedron

The polyhedron has four potential corner rotations. For each corner rotation twelve limit planes define the rotation space; six limit planes for the rock mass corners ***C4*** and ***C7***, as well as six limit planes for the moved corners at the free face. In total, the rotation space is formed by 48 limit planes. The vectors pointing from the corners to the rotation axis are not only defined by the intersection of the block planes. It is necessary to determine all vectors pointing from the corners to the static point on the rotation axis. Since this requires determining the block geometry, it cannot be solved using the stereographic projection alone. For the solution the direction of the vectors ***C1-C6*** and ***C2-C5*** at the free face, as well as ***C4-C5***, ***C4-C6***, ***C2-C7*** and ***C1-C7*** within the block is required.

Figure 5 shows the stereographic projection of the joints and free face of the polyhedron including their poles, as well as the joint pyramid and the rotation space. By comparing space pyramid and joint pyramid, removability of the block is determined. However, according to Mauldon (9) this is not a necessary condition for blocks with more than three joint planes. The block is corner-rotatable about axes through ***C1***, ***C2*** and ***C5*** which plot inside the corresponding rotation space. The block is also edge-rotatable about edge ***C1-C5*** since ***E15*** forms part of the rotation space for corner ***C1*** and ***C5***. In contrast to Figure 4 the rotation space is significantly decreased for the original block cut by a daylighting joint plane.

Tetrahedron

Since this tetrahedron consists also of three joints and one free face, it has three potential corner rotations. Rotatability can also be judged by 21 limit planes.

Figure 6 shows joints and free faces including poles, as well as the joint pyramid and rotation space. The block is removable (necessary condition). About the corners ***C3*** and ***C6*** the block is rotatable. Although the possible dip angle of rotation axes for ***C3*** rotation varies only by approx. two degrees, under certain loading conditions it can be critical. Finally, the block is not edge rotatable. Observing Figure 6 it seems that the edge ***E65*** plots inside the rotation space for ***C6***. Since the rotation space for ***C5*** is empty, this would be an inconsistent result (Edge rotation requires corner rotatability for both corners forming the edge). A closer examination preserves consistency: ***E65*** forms part of joint plane 4. The rotation space for ***C6*** is delimited by limit plane $\boldsymbol{n_1}\times(\boldsymbol{n_4}\times\boldsymbol{n_1})$ which is almost parallel to joint plane 4. In the sector of the rotation space for ***C6*** the limit plane plots in the upper halfspace of joint plane 4. Since the rotation space for ***C6*** is also

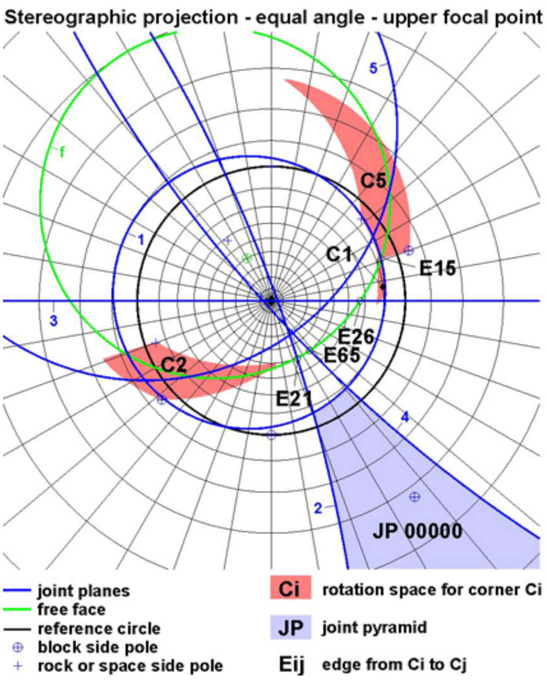

Fig. 7 Stereographic projection of joint planes, free face, joint pyramid and rotation space for the polyhedron $U_1U_2U_3L_4L_5L_f$.

Bild 7 Stereographische Projektion der Kluftflächen, freien Oberfläche, Kluftpyramide und des Rotationsraums für das Polyeder $U_1U_2U_3L_4L_5L_f$.

defined by the upper halfspace of the limit plane, the edge ***E65*** is not contained in the rotation space. Therefore, the block is not edge rotatable.

Intersection with a non-daylighting joint plane

If the original block is cut by joint planes 4 and 5, another convex polyhedron is generated. The polyhedron $U_1U_2U_3U_4U_5L_f$ consists of the corners ***C1***, ***C2***, ***C5*** and ***C6*** at the free face, as well as ***C7***, ***C8***, ***C9*** and ***C10*** inside the rock mass. Note that joint plane 5 does not daylight. For any of the four corners at the free face 18 limit planes have to be determined; twelve for the four rock mass corners and six for the three moved corners at the free face. In total 64 limit planes delimit the rotation space. Again, the limit planes can only be determined if the block geometry is known.

Figure 7 shows the stereographic projection of the situation of this block. Since joint plane 5 does not daylight, JP00000 remains the same as JP0000. Therefore, the block is removable. The rotation space is non-empty for corners ***C1***, ***C2*** and ***C5***, hence, the block is corner rotatable about them. Edge ***E15*** forms part of the rotation spaces for ***C1*** and ***C5***, hence, the block is also edge rotatable about ***E15***. Although the joint pyramid has not changed, it can be observed that the rotation space has increased for corner ***C5*** (compare with Figure 5). This fact is critical for rotational kinematics of blocks. Non-daylighting joint planes increase the kinematical freedom and can change non-rotatable blocks to rotatable ones.

Conclusion

A method for the analysis of the kinematical freedom of rock blocks to rotation has been presented. This method allows determining the rotatability of block to the pure rotation modes *corner rotation* and *edge rotation*. It is suitable for every block geometry; convex or non-convex polyhedra. The result of the analysis is the rotation space which indicates all direction of rotation axes which cause kinematically admissible displacements of the block. In an illustrative example the application of this method has been demonstrated. The results have been presented in hemispherical projection plots. In this example the influence of daylighting and non-daylighting joint planes on the rotatability has been examined. It can be stated that non-daylighting joint planes lead to an increase of the kinematical freedom. More critical, if their influence is not considered, the rotatability potential is underestimated. They can change a block from being non-rotatable to rotatable.

This method cannot be considered as a complete stability analysis, but it forms the basis for subsequent determination of the failure modes, and the stability assessment. Every block analysis should start with the analysis of kinematics. Only kinematically free (removable and/or rotatable) blocks merit further analysis of failure modes and stability. On the other hand, kinematically restricted blocks (non-removable and non-rotatable) can be directly considered as stable. The method can be directly applied to rock slope engineering where block failure mechanisms play a dominating role.

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Authors

Dipl.-Ing. Markus Pötsch, o.Univ.-Prof. Dipl.-Ing. Dr.mont. Wulf Schubert, Institute for Rock Mechanics and Tunnelling, Graz University of Technology, Rechbauerstraße 12, A-8010 Graz, Austria, email: m.poetsch@tugraz.at; schubert@tugraz.at