DK 528.721.085.32(430–313.5):771.536:512.52

OEEPE Oberschwaben Reseau Investigations

(with 10 figures, 13 tables)

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RÉSUMÉ see p. 316

1. Introduction

In the framework of the larger experiment on aerial triangulation, the OEEPE wished to include an additional experiment with reseau photography. To that end, the five alternate strips of the non-reseau wide-angle block Oberschwaben, that had been photographed with 90 % forward overlap, viz. strips 6, 8, 10, 12 and 14, were re-photographed with the Zeiss Reseau camera RMK AR 15/23, also at the scale 1 : 28,000 and again with 90 % forward overlap, and with 20 % side lap.

ITC, which acted as a pilot centre for Commission F of the OEEPE (i. e. the commission that deals with fundamental problems in photogrammetry), two investigations have been carried out with the above reseau photography. The first investigation concerned monocular reseau observations in some 37 glass positives, the second investigation concerned reseau observations during stereo-restitution on the Wild Stereocomparator of glass positives of some 100 stereomodels. The results of these two investigations, as carried out in 1970–71, were published in Photogrammetria, 27 (1971), pages 169–199. The main findings of these investigations are given in section 2.

Meanwhile, at the Photogrammetric Institute of Stuttgart University, West Germany, *K. Kraus* investigated the method of linear least squares interpolation, as originally developed some 30 years ago, and further developed by *H. Moritz* for application in gravimetry (1970). *Kraus* applied this method to photogrammetric interpolation problems, including the problem of correction for film deformation using reseau photography.

To that end, he used the ITC monocular observations of three OEEPE reseau plates (nos. 302, 358 and 412 of strip 10) and the ITC stereo observations on two stereomodels, formed by another three OEEPE reseau plates, covering the test field Rheidt (nos. 39, 43 and 48).

The results of the linear least squares interpolations were published in Photogrammetric Engineering, May 1972, pages 487–493 (as for the monocular observations) and in Photogrammetria, 28 (Nov. 1972), pages 179 and 180 (as for the stereo observations). The main findings of these two investigations are given in section 3.

At ITC, subsequently, the authors of the present paper rounded off the investigations by including somewhat less superior photo material in their research. Sofar, the OEEPE Oberschwaben photography had shown to be "too good", from the geometrical point of view, to improve it significantly by the use of the reseau. To obtain less superior material, ITC ordered, at the end of 1972, besides new glass diapositives of the original negatives (then taken $3\frac{1}{2}$ years ago), also positives on (polyester-)film.

After observing the film positives, the transparancies were "mistreated" by placing them on a heated glass plate and subsequently were re-observed. The results of these observations are presented in section 4.

2. The ITC Reseau Investigations 1970–1971

2.1. Monocular Reseau Investigation

In the central strip, run 10, 25 plates were selected thus forming a strip with approximately 60 % forward overlap. Three plates were selected at the beginning, middle and end of each of the other four strips.

The monocular reseau measurements were carried out mainly on the Wild STK 1 Stereocomparator, using a black circular measuring mark with a diameter of 60 microns.

All 524 reseau crosses were observed in 5 plates of strip 10. In the other 32 plates an irregular 2cm- and 3cm-grid of 81 points was measured, in three instances both backwards and forwards in order to estimate the pointing accuracy. The distribution of these 81 points can be seen in the vector diagrams, figures 4–6. The standard deviation of a single pointing appeared to be:

$$\sigma_x = 2.0\mu \; ; \quad \sigma_y = 3.1\mu.$$

The monocular reseau measurements were transformed, by linear conformal transformation, onto the ideal grid of the 81 points. The 37 vector diagrams of residual distortions, obtained after conformal transformation of each of the 37 plates showed:

a. that the residual distortions were fairly small: very small in the central square of 12 x 12 cm; outside this square the order of magnitude of the vectors is some 7 microns, with maximum values of 20 to 24 microns along the edge of the photographs. See figures 1–6*).

b. that the distortion pattern was fairly systematic throughout a strip. In order to show this in numerical form, average distortion values were computed per point (81 points), for each strip. In the case of strip 10, the averages could thus be determined from 25 plates (and in the other strips from 3 plates each). Comparing now, per point, the distortion values in each of the 25 plates with the corresponding average values, a "noise" of 2.6 μ in x, and 2.8 μ in y, is found. For the other four strips, similar values were found. Bearing in mind the accuracy of the pointing error (σ_x = 2.0 μ; σ_y = 3.1 μ), it is clear that the variation of the distortion pattern within a strip is insignificantly small.

c. that, although the distortion patterns remained fairly constant throughout a strip, and to some extent between adjacent strips, there was a marked difference between the patterns of the two outer strips 6 and 14.

In order to find the minimum number of reseau points, and the optimum distribution of these points, as required for correcting the systematic components of the film deforma-

*) In the present paper, only some of the figures as published in Photogrammetria 27 (1971) are included.

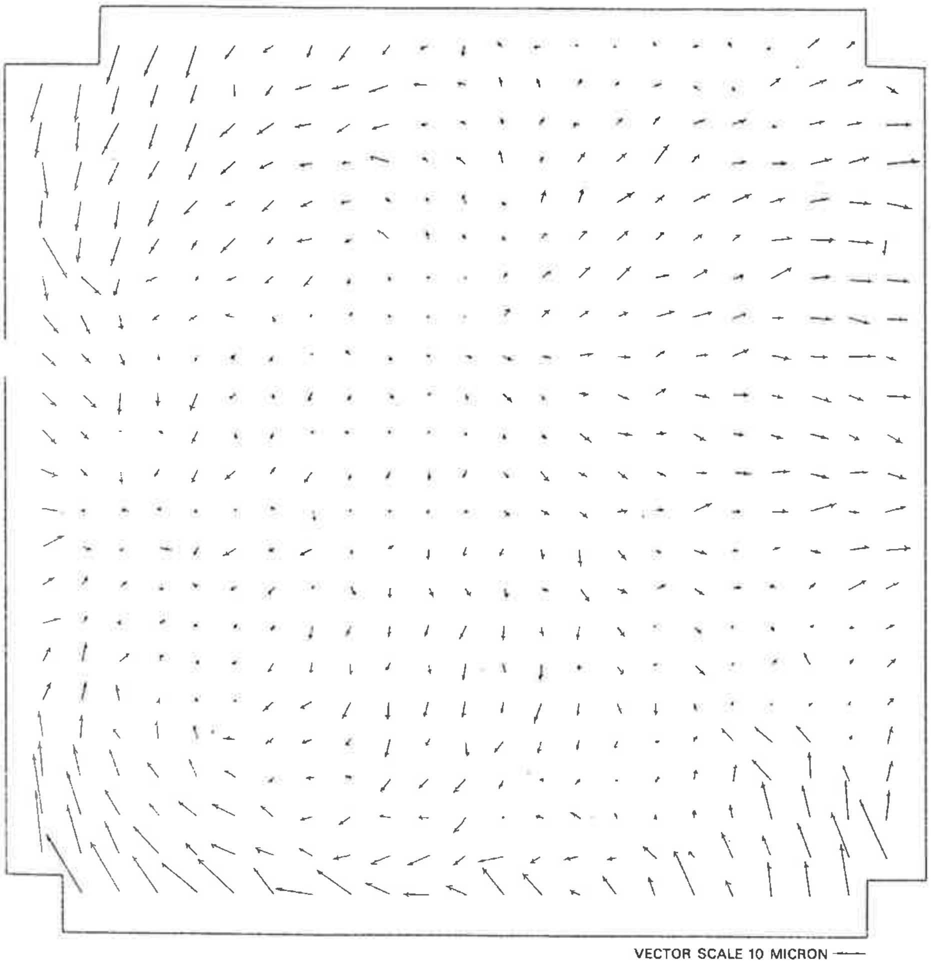


Figure 1 – Residual distortion-plate 302

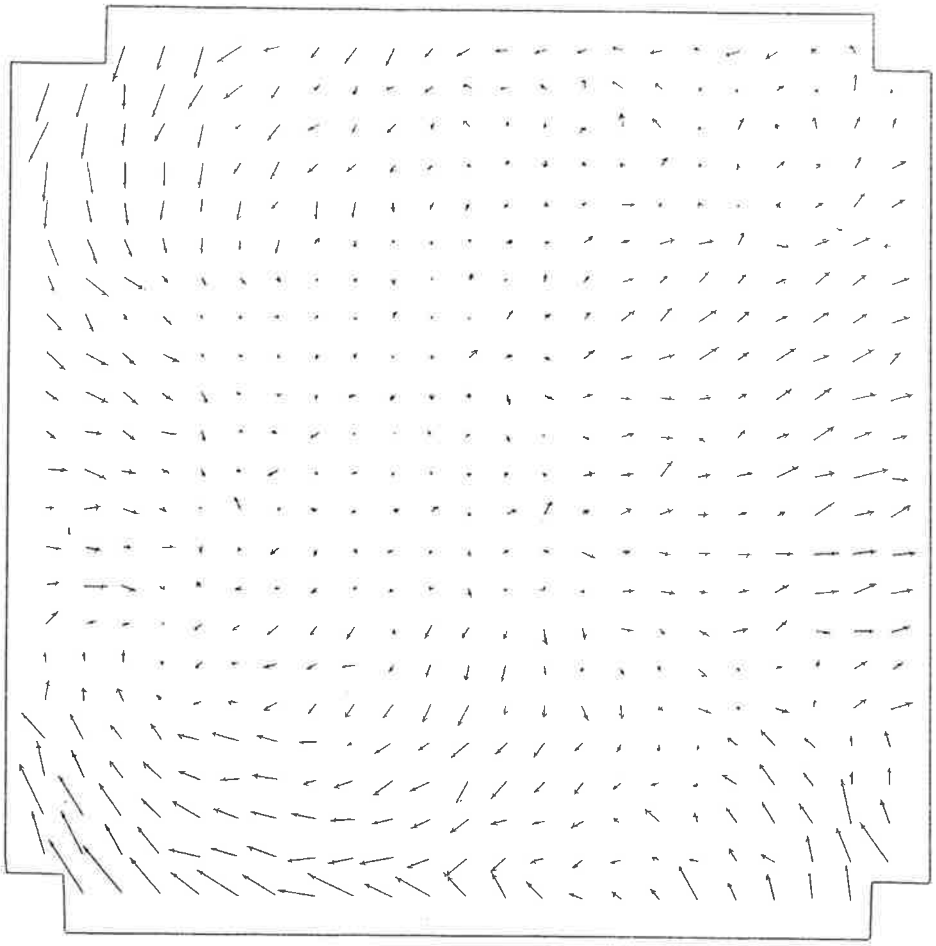


Figure 2 – Residual distortion-plate 358

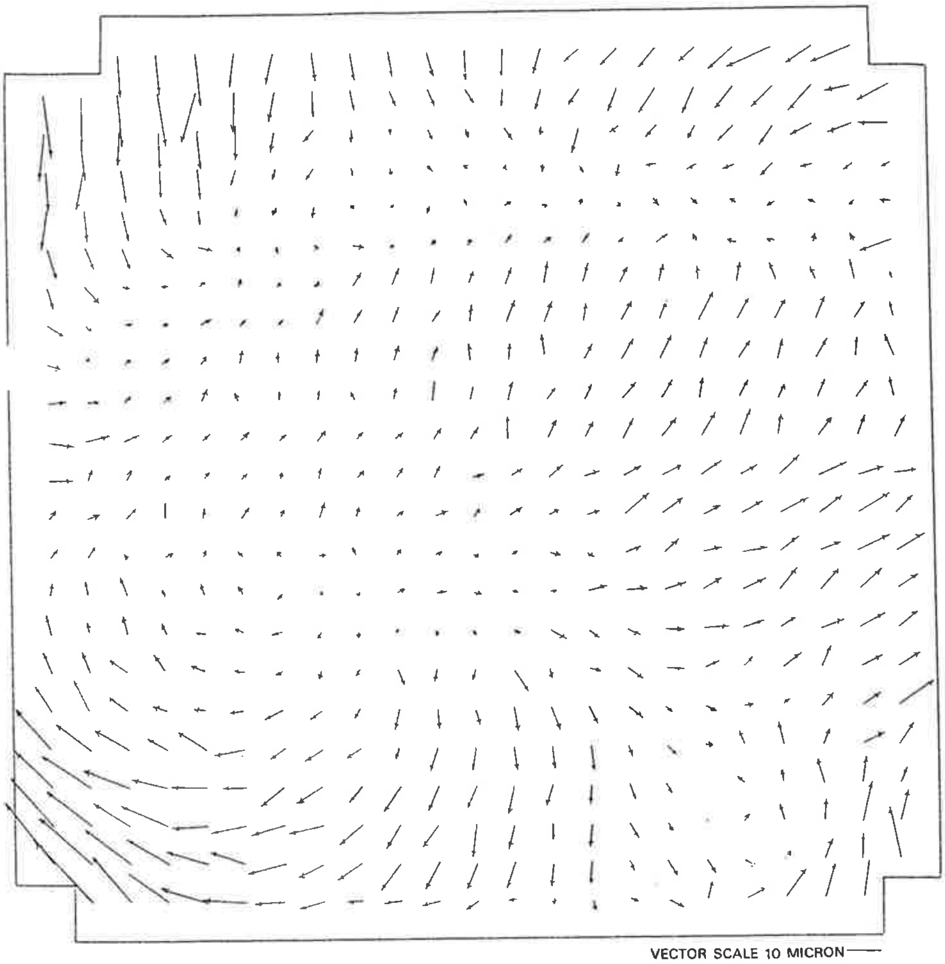


Figure 3 – Residual distortion-plate 412

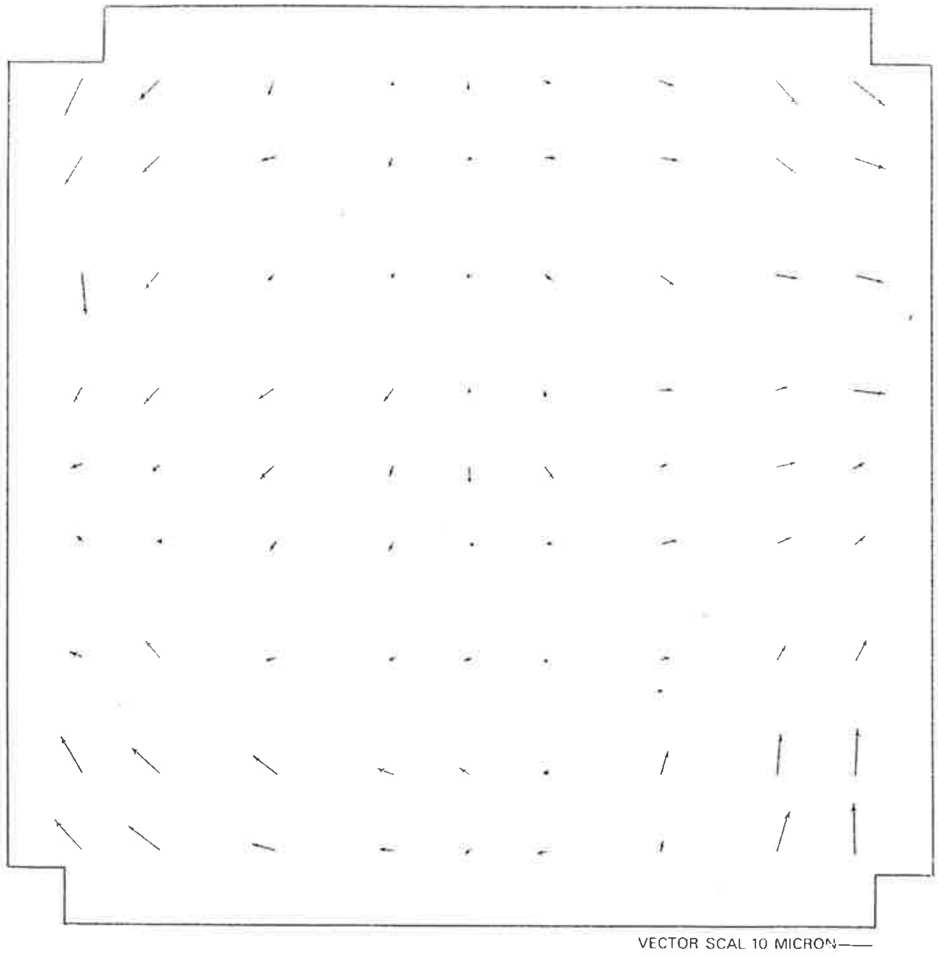


Figure 4 — Average distortion-strip 6

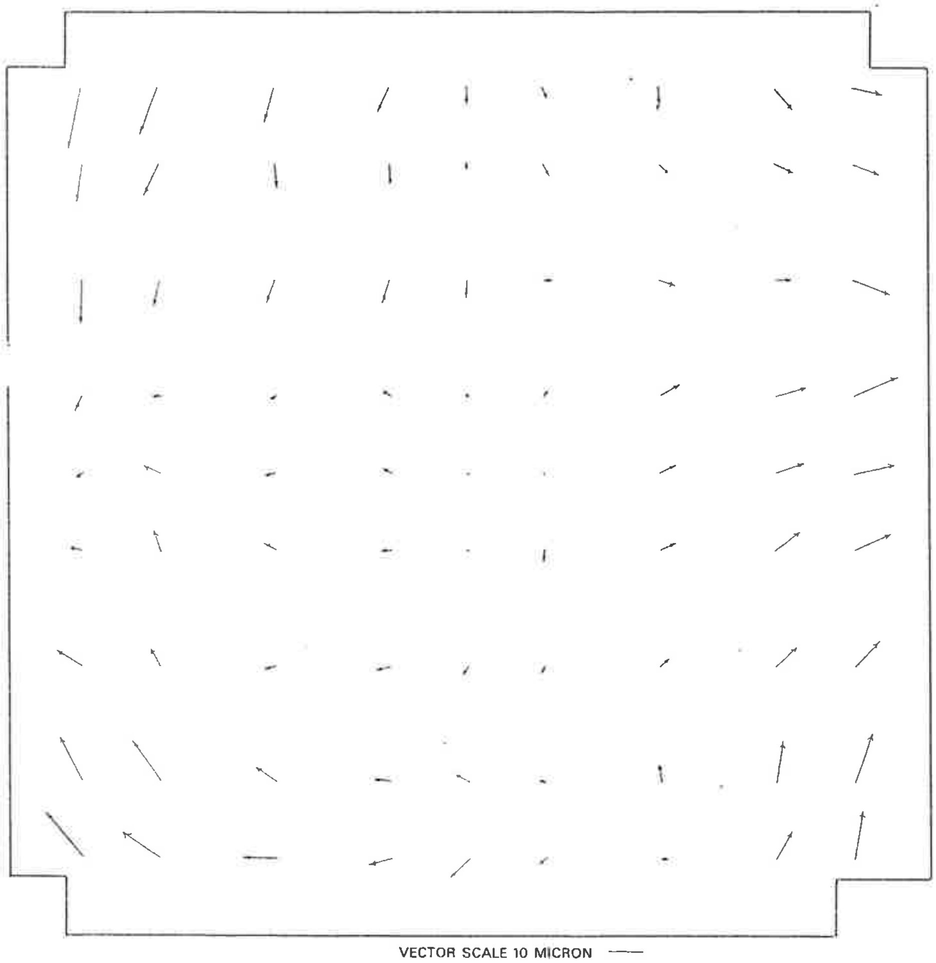


Figure 5 – Average distortion-strip 8

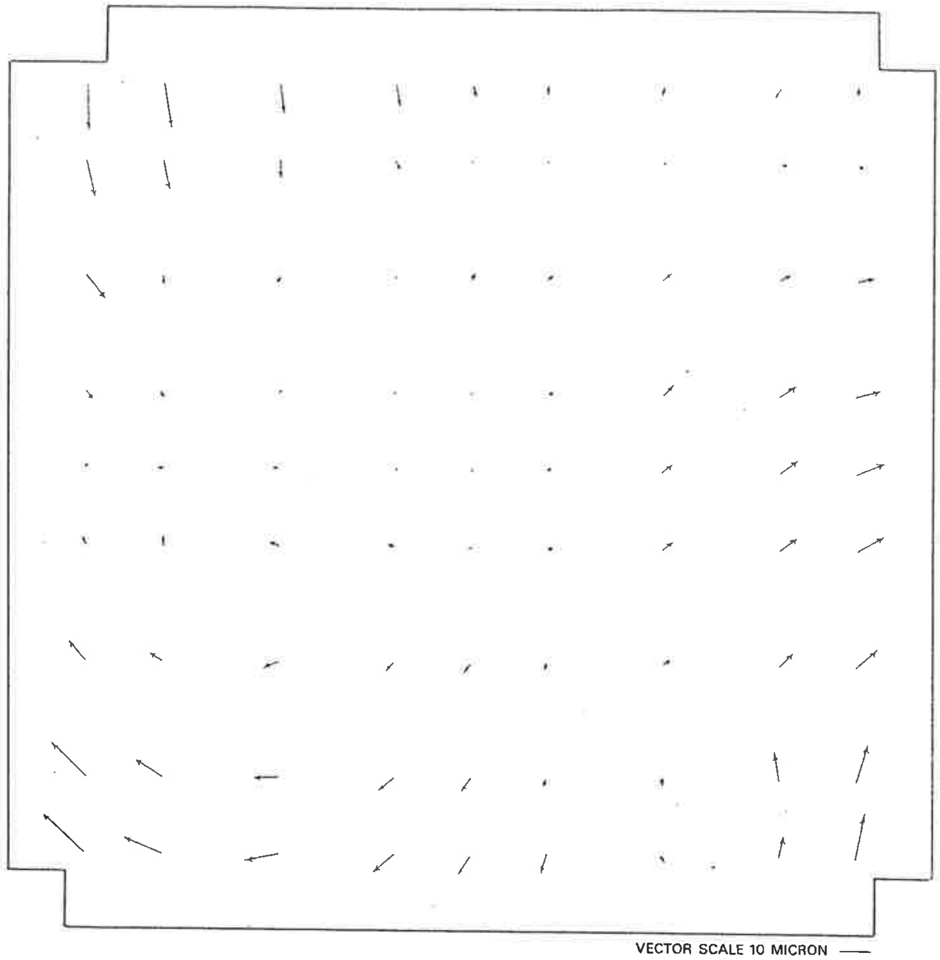


Figure 6 – Average distortion-strip 10

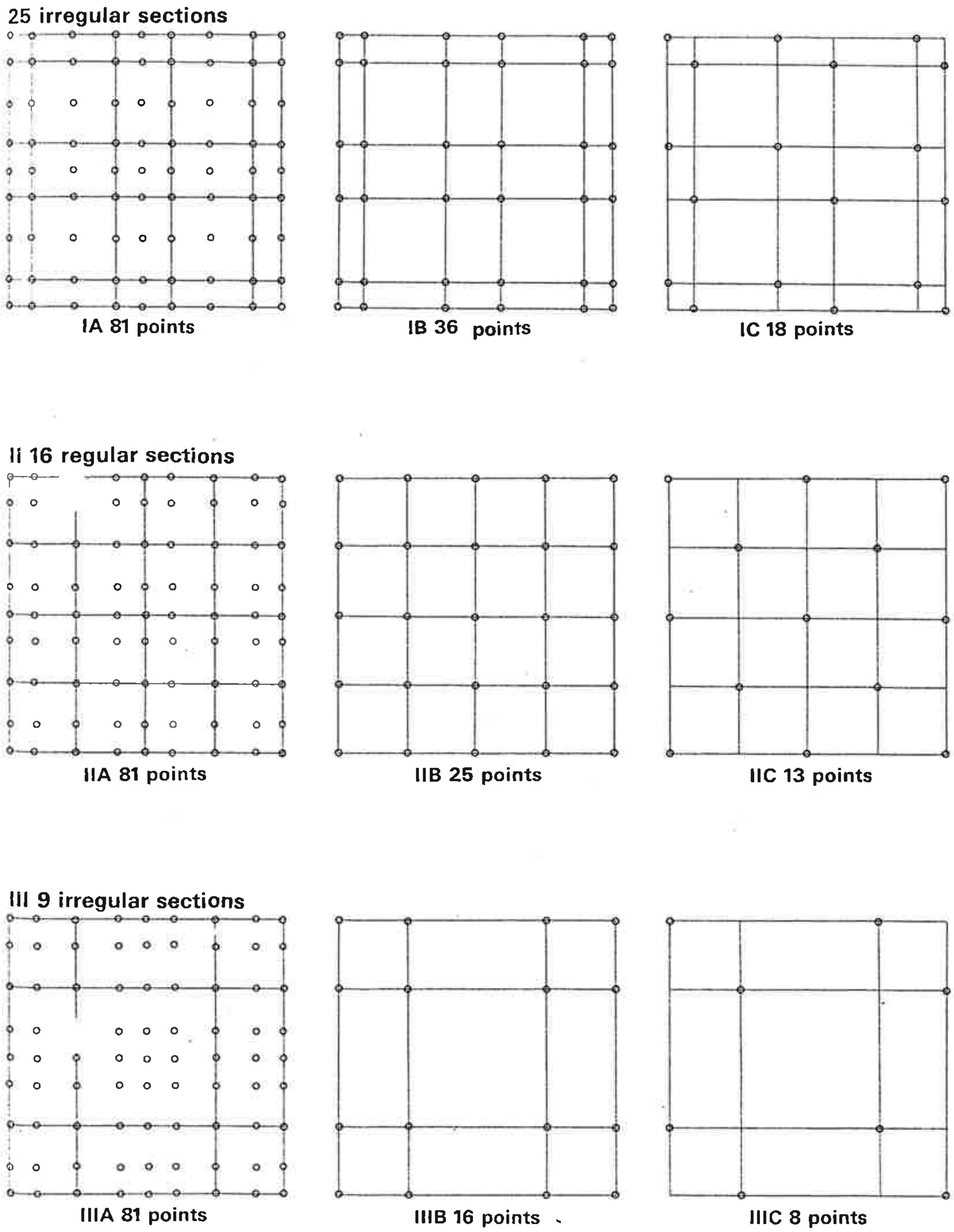


Figure 7 – Division of plate into sections

Table 1 – Section means per strip, in microns

Sect.	x-component					y-component				
	St. 6	St. 8	St. 10	St. 12	St. 14	St. 6	St. 8	St. 10	St. 12	St. 14
1	– 0.3	– 1.6	– 0.5	– 8.6	– 5.9	– 12.9	– 12.1	– 10.2	– 14.2	– 9.6
2	0.6	0.4	1.5	– 2.6	– 2.5	– 5.7	– 4.8	– 1.6	– 1.7	0.8
3	3.4	3.8	3.1	3.0	1.3	– 0.6	0.6	4.2	6.7	6.6
4	6.7	7.9	7.5	11.9	8.8	– 4.0	– 3.8	– 0.3	1.9	0.2
5	7.3	8.4	9.5	16.6	12.2	– 8.1	– 9.7	– 7.0	– 4.4	– 6.0
6	2.6	4.1	4.4	10.2	7.0	– 5.3	– 8.9	– 5.7	– 7.1	– 9.0
7	2.0	3.7	2.4	5.5	3.6	– 1.3	– 3.0	0.4	– 0.7	– 2.8
8	1.1	2.9	1.6	1.5	1.7	– 0.8	1.5	1.9	0.7	– 2.3
9	– 1.7	– 0.4	0.0	– 4.1	– 2.1	– 6.0	– 3.9	– 4.0	– 7.7	– 8.2
10	– 2.3	– 5.5	– 3.0	– 7.8	– 6.8	– 7.5	– 8.0	– 6.2	– 11.6	– 9.6
11	– 3.8	– 9.1	– 6.0	– 7.9	– 6.6	– 2.1	– 3.4	– 3.7	– 9.3	– 7.2
12	– 3.2	– 4.0	– 3.6	– 4.2	– 1.9	– 0.5	0.6	– 1.5	– 5.4	– 5.8
13	0.0	1.4	– 0.5	– 0.6	1.3	1.9	1.0	0.3	– 2.9	– 3.2
14	0.8	2.1	0.3	3.3	3.2	1.4	– 3.4	– 1.1	– 4.4	– 3.7
15	0.7	1.9	– 0.9	1.4	3.2	1.7	– 0.7	0.0	– 1.9	– 1.9
16	3.5	3.0	– 1.7	– 2.3	– 1.1	4.4	6.1	5.2	5.9	7.0
17	3.6	3.2	– 0.5	– 0.4	– 1.2	4.1	3.4	4.1	3.5	5.3
18	– 0.5	0.2	– 0.2	0.2	– 0.1	2.0	1.2	0.4	1.0	0.7
19	– 3.7	– 5.2	– 2.7	– 3.4	– 3.3	– 0.4	0.8	– 1.4	– 1.5	– 1.8
20	– 6.3	– 8.0	– 3.2	– 1.5	– 2.7	0.9	0.4	– 1.1	– 1.4	– 2.9
21	– 7.1	– 6.2	– 0.1	3.3	0.0	4.6	4.5	1.7	7.9	3.3
22	– 4.5	– 3.4	0.3	1.4	– 0.5	3.2	4.9	1.5	7.7	4.4
23	– 1.6	0.6	– 0.9	– 1.8	– 3.6	1.0	5.4	3.1	6.9	4.6
24	2.5	3.6	– 1.3	– 2.5	– 4.7	3.2	7.5	6.7	9.4	9.1
25	4.7	3.8	– 0.9	– 3.8	– 6.3	7.1	12.3	10.3	13.3	15.3
Mean	3.0	3.8	2.3	4.4	3.7	3.6	4.5	3.3	5.6	5.3

tion, the plate was sub-divided into sections. The average distortion within each section was denoted as the systematic distortion component and the variation within each section as the random component. In this experiment, three main section divisions were examined, whereby the 20cm-square-grid was divided into respectively 25, 16 and 9 sections (see figure 7), while the following point distributions were tested:

25 sections:	81, 36 and 18 points
16 sections:	81, 25 and 13 points
9 sections:	81, 16 and 8 points

Using the average plate of strip 10, the section means were determined separately for the 9 section divisions, and the variations within each section were computed. From the results, the following conclusions can be drawn:

d. for a given number of sections (either 25, 16 or 9), the number of points used to define the systematic component is not significant. Maximum vector errors of 2.8 μ, 1.8 μ and 2.5 μ, respectively, were found between the use of 81 or 18 points (in the case of 25 sections), 81 or 13 points (16 sections), 81 or 8 points (9 sections), respectively, to establish the section means.

e. it is obvious that the "random component" will increase with an increase in the size of the section, but the differences found in this investigation between using either 25, 16 or 9 sections were hardly significant. For the minimum number of points used for each case (18, 13 and 8 points, respectively), the maximum vector errors increased from 7.1 μ (25 sections) to 11.30 μ (9 sections).

Under point c above, mention was made that, although the distortion patterns remained fairly constant throughout a strip, there was some variation from strip to strip, and particularly between the two outer strips 6 and 14. In order to show this in numerical form, the section means were computed, for a division into 25 sections and using 18 points, for each strip, and these means were compared in table 1. From table 1 it can be seen that there are differences up to 11 μ between section means computed from average plates of separate strips and that it is thus better not to adopt an average distortion pattern for the whole block.

To conclude this investigation, it was decided to simulate a practical example whereby the residual distortion would be determined for a number of plates in the strip, say at the beginning, middle and end, followed by the computation of the average distortion per point (18 points) and per section (25 sections). These section means would now be considered indicative of the systematic component for all plates in the strip, and it then remained to be seen whether the residual random components of individual plates are tolerable or not.

Strip 10 was selected since its section means as given in table 1 were obtained from the point averages of 25 plates. The standard deviations of the random components of three plates were as follows:

Table 2

Strip	Plate	n	Stand. Dev.	
			x	y
10	302	169	3.0 μ	3.2 μ
10	362	169	2.9	3.8
10	412	169	2.8	3.7
10	Total	507	2.9 μ	3.6 μ

In assessing these results, they should be compared with the 2.6 and 2.8 μ found above (under point b) for the difference (the "noise") between the distortions in individual plates of strip 10, and the average distortions, using 81 reference points rather than 18.

Conclusion: the rather small decrease in accuracy obviously justifies this sub-division into sections and the adoption of the section means as representative of the distortion within that local area.

Recommendations on the use of the reseau: (1) Before commencing with an aerial triangulation, select at least three plates in each strip, preferably at the beginning, middle and end, and measure a limited but selected number of reseau points in each plate (i e., see figure 12, where 8, 13 or 18 points could be measured depending on whether one desires 9, 16 or 25 sections, respectively). (2) The analyses of these three plates per strip will either confirm that the distortion is small and uniform, in which case the reseau need not be used at all during the triangulation phase, or that the distortion is significant and irregular and that it must be corrected. In the latter case, one will first have to examine whether the division into sections is sufficient – this will automatically yield the minimum number of points required to define the systematic component within each section, and only this limited number of points need then be measured in each plate during the triangulation phase.

2.2. Use of Reseau in Analytical Stereo-restitution

In his paper as presented at the Image Deformation Conference, June 1971, Ottawa (Sources of Image Deformation, Photogrammetric Engineering 1971, pages 1259–1265), *H. Ziemann* describes all sources of image deformation. *"A number of examples are given for deformation of unusual size caused by different sources. These examples demonstrate that a dense centrally projected reseau exposed simultaneously with the original negative is the only solution that permits checking the image deformation at any time caused by any source. A reseau can also be utilized to correct any deformation of the image to an extent limited only by its density, if analytical procedures are used"*.

A systematic procedure whereby the distortion in each observed point in each individual photograph is located, and corrected for, will of course require some more effort in the observational, as well as in the computational, procedure.

In analytical stereo-restitution, either of single models (on behalf of numerical determination of terrain coordinates of selected points such as signalized cadastral boundary corners, etc.), or of strips and blocks (in aerial triangulation), points are observed on a comparator.

In the case of aerial triangulation on a stereocomparator, the number of points to be observed per stereo-model is usually:

monocular observations: 2 x 4 fiducial marks

stereoscopic observations: 6 orientation points, 6–8 minor control points, 2 x 2 scale-transfer points.

According to the statistics, collected at ITC from analytical triangulation of more than 5,000 stereo-models, the observation time on a stereocomparator is about 45 minutes per stereo-model (including teabreaks, etc., etc.).

In order to be able to trace, and to correct for, distortion errors in the points observed, immediately after the usual stereoscopic observation of a point, the nearest reseau cross : each of the two photographs may be observed (monocularly).

For highest accuracy, in each photograph the four reseau crosses that surround the observation point concerned, should be measured and the distortion of the point be interpolated from the distortions found in the four crosses. As, however, with a centimetre-reseau, the nearest cross is never more than 5 mm away in each coordinate direction, we considered it sufficient to observe in each photo only the nearest cross.

The additional time for the monocular observations of the nearest cross in each photo has appeared to be 27 seconds per point observed; for a normal stereo-model in the case of aerial triangulation thus an additional 7 $\frac{1}{2}$ minutes is required.

Experiments

In ITC, analytical aerial triangulation has been carried out on the Wild Stereocomparator of three of the four sub-strips of models with 60 % forward overlap, that could be formed from the photographs of strip 10 which had 90 % forward overlap: strips 10–1, 10–3 and 10–4, respectively, with 24, 23 and 22 models, respectively. The triangulation was carried out with contact diapositives on glass.

The measuring and computation programmes of the ITC Stereo-Reseau method have been cted in such a way that all reseau-cross observations can be eliminated from the original observations. Each stereo-model etc. can thus be computed with or without use of the reseau observations.

Results after strip-adjustment according to *Ackermann*:

Table 3 – Short bridging distances

Strip	Total nr. of models	Nr. of models bridged between ground control		With use of reseau crosses			Without use of reseau crosses		
		Plan	Height	Plan (cm) X	Plan (cm) Y	Height (cm)	Plan (cm) X	Plan (cm) Y	Height (cm)
10–1 first part	9	1.5	1.5	22	22	51	24	28	40
10–1 sec. part	9	1.5	1	19	15	36	18	18	23
10–3	11	2	2	27	29	30	23	23	46
10–4	14	3.5	2	32	34	53	28	26	69
				25	25	42	23	24	45

Conclusions: no significant difference between the two cases.

For these very short bridging distances:

$\sigma_X = \sigma_Y$ = 24 cm = 9 microns at the scale of photography

σ_H = 43 cm = 0.10 $^0/_{00}$ of the flying height

Table 4 – Longer bridging distances

Strip	Total nr. of models	Nr. of models bridged between ground control		With use of reseau crosses			Without use of reseau crosses		
		Plan	Height	Plan (cm) X	Plan (cm) Y	Height (cm)	Plan (cm) X	Plan (cm) Y	Height (cm)
10–1 first part	9	4.5	4.5	25	28	64	28	32	55
10–1 sec. part	9	4.5	4.5	24	26	49	25	25	26
10–3	11	5.5	5.5	34	33	67	28	31	87
10–4	14	7	7	33	39	49	27	26	75
				29	32	57	27	29	61

Conclusions: no significant difference between the two cases.

For these longer bridging distances:

σ_X = 28 cm, σ_Y = 30 cm = 10 microns and 11 microns respectively, at the scale of photography

σ_H = 59 cm = 0.14 $^0/_{00}$ of the flying height

Absolute accuracy of numerical restitution.

The same camera was used to photograph the test field Rheidt (close to Bonn) at a scale 1 : 10,500, both before and after taking the reseau photography of the Oberschwaben block.

Some models of this test field were observed on the Wild Stereocomparator. Each model was oriented absolutely with the help of 14 control points. With some 50 further check points, the results as shown in table 5 were found.

Table 5 – Absolute accuracy of numerical restitution

Model	With use of reseau crosses	Without use of reseau crosses
39/43	σ_X = 0.09 m = 9 μ σ_Y = 0.08 m = 8 μ σ_H = 0.13 m = 0.08 ‰ Z	σ_X = 0.08 m = 8 μ σ_Y = 0.08 m = 8 μ σ_H = 0.16 m = 0.10 ‰ Z
43/48	σ_X = 0.08 m = 8 μ σ_Y = 0.07 m = 7 μ σ_H = 0.12 m = 0.075 ‰ Z	σ_X = 0.07 m = 7 μ σ_Y = 0.09 m = 9 μ σ_H = 0.12 m = 0.075 ‰ Z

Conclusion: no significant difference between the two cases

The reseau, of course, can only be used for the location and correction of the distortion of the aerial photographs that is caused by an unflatness of the emulsion at the moment of exposure, by film distortion and by the photographic processing of the negatives and positives. Lens errors and refraction cannot be corrected for by the use of the reseau.

The above results show that, in the Oberschwaben reseau photography, the distortion due to unflatness, film distortion and photographic processing is so small that the improvement of the geometry of the images, effected by the correction for that distortion in each point observed, is just compensated by the additional errors in the pointing to the reseau crosses (together with the variation in the distortion over distances up to 5 mm).

3. Reseau Investigations at Stuttgart University

3.1. *K. Kraus* (Phot. Eng. 1972) introduced a new definition of the irregular (or random) and systematic components of the film deformation. Contrary to the analysis described in section 2.1 above, the new definition works without subdividing the photograph into smaller units.

Kraus states that, "*in the description of a vector field, the following items are of particular interest:*

– *Absolute magnitude of the vectors*

– *Separation of systematic and irregular components*

– *The range of systematic effects and their variation as a function of distance.*

It is assumed that no correlation occurs between the x and y component of the two-dimensional vectors. We then search for concepts or terms describing mathematically the three items, separately for x- and y-directions.

The absolute magnitude of the reseau image errors l_i in the x- or y-direction can be described by the variance $V = M\,[l_i\, l_i]$, where M means mean square value. The variance contains the irregular as well as the systematic components of the vector field.

We also introduce the covariance $C\,(s_{ik}) = M\,[l_i\, l_k]$, which is the mean-square value of the cross product of the errors $l_i\, l_k$ of all reseau points P_i and P_k. The covariance contains the systematic part of the vector field, the irregular part being filtered out. The empirically determined covariances (for some groups of approximately equal distances s) are a perfect indicator on how the common systematic effects between the points decrease with increasing distance between them.

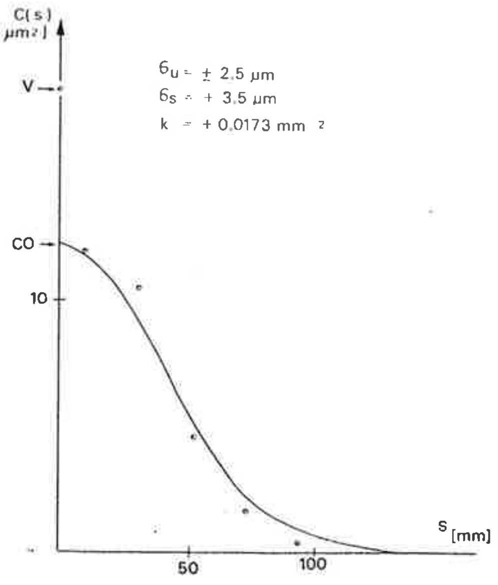


Figure 8

Figure 8 shows first the variance V and the empirical covariances $C\,(s_{ik})$ for a number of distances s of the y-component of the vector field of measured values of reseau points of

Plate 358, Oberschwaben. The empirical covariances can be approximated by a continuous function, the covariance function, for which the well-known Gaussian curve is the most appropriate:

$$C(s) = CO. \quad Exp\left\{- k^2 \cdot s^2\right\}.$$

The maximum value CO is the covariance between points which are infinitely close together (s = o). The constant k is responsible for the rate of decrease of the covariance function with increasing distance.

The conclusion is that three constants are sufficient to describe the behaviour of the components of a vector field. These are: the variance V, the maximum value CO of the covariance function, and the constant k.

For the y-components of the vector field of Plate 358, we found the values:

$$V = 18.4\ \mu^2 \quad CO = 12.2\ \mu^2 \quad k = 0.0173\ mm^{-2}$$

From these parameters the numerical values for the three terms by which we describe the three basic features of a vector field can be derived directly:

the mean square value of the absolute magnitude of the y-components was:

$$\sigma y_a = \sqrt{V} = 4.3\ \mu$$

- *the mean square value of the systematic component was:*

$$\sigma y_s = \sqrt{CO} = 3.5\ \mu\ ,$$

while the mean square value of the irregular component was:

$$\sigma y_u = \sqrt{V - CO} = 2.5\ \mu$$

- *the constant k = 0.0173 is a representative value for the behaviour of the systematic effects depending on the distances."*

For the linear least-squares interpolation, for the details of which one is referred to Phot. Eng., Oct. 1972, pages 1016–1029, it is exactly the above 3 constants that are required. Important practical features of the method are:

- the method is independent of the number and distribution of the reseau points observed;
- the irregular errors of measurement, together with the short period variations in the distortion over distances of some millimeters only, are not transferred to the image points which have to be corrected (interpolated), but instead are filtered out.

3.2. The method was applied on the monocular reseau observations, carried out in ITC, of Plate 358 of strip 10 (with 524 reseau points observed). The problem that was researched, was the problem of how many reference points (reseau point observations) were needed to correct the systematic component of the film deformation.

To that end, firstly 147 reseau points (distributed as a 2cm-grid), then 52 (a 4cm-grid) and next 25 (a 5cm-grid) were used as reference points, and the other points interpolated. All three distributions proved to be very effective, the latter one only very slightly less than the former.

Next, reference points were selected along the edges of the photograph only, these points thus are equivalent to fiducial marks. It then appeared that, for the photograph concerned (Plate 358), the systematic film deformation could be caught rather well in the case of using 27 reference points along the edges. However, when using only 8 or, still worse, 4 reference points only, the results were very poor. *"The conclusion is (thus) justified that the conventional fiducial marks in air survey cameras are not suited to correct the systematic film deformation"*.

3.3. The method was also applied on the measurements on the two test-models of Rheidt, executed at ITC. The results are shown in the following table 6 in which the earlier results of table 5 (section 2.2 above) are added in brackets.

Table 6

Model	With use of reseau	Without use of reseau *)
39/43	$\sigma_X = 6.2\,\mu$ (8.8) $\sigma_Y = 5.8\,\mu$ (8.0) $\sigma_H = 0.075\ ^0/_{00}\ Z$ (0.08)	$\sigma_X = 7.8\,\mu$ (7.8) $\sigma_Y = 7.2\,\mu$ (7.9) $\sigma_H = 0.097\ ^0/_{00}\ Z$ (0.10)
43/48	$\sigma_X = 6.8\,\mu$ (8.0) $\sigma_Y = 5.5\,\mu$ (7.0) $\sigma_H = 0.064\ ^0/_{00}\ Z$ (0.075)	$\sigma_X = 6.6\,\mu$ (7.2) $\sigma_Y = 6.0\,\mu$ (9.1) $\sigma_H = 0.071\ ^0/_{00}\ Z$ (0.075)

*) The small differences between corresponding values are primarily caused by the fact that earlier only 14 points were used for the absolute orientation, while Kraus used all ground check points.

"Earlier a method was used by which the measured value of the closest reseau point is directly transferred to the image point to be corrected. While that method caused no improvement, the present method shows a significant improvement of the accuracy. For practical applications, the improvement of 20 % of model 39/43, which originally was less accurate, is especially important.

According to these results, the reseau in combination with least-squares interpolation seems to yield 7 μ in planimetry and 0.075 $^0/_{00}$ Z in height, which is independent of the magnitude of the original film deformation".

4. Latest Experiments

4.1. Introduction

The experiments have been concluded at the ITC with new glass diapositives of the original negatives, with positives on polyester film and also with "artificially aged" film positives.

4.2. Stereoscopic Measurements

The abbreviations used to denote the different variants of the two test-models of Rheidt, nrs. 39/43, 43/48 as obtained from the new diapositives, are summarised in table 7.

In the models GL, FI, AA and AB, the floating mark used was a dot of ϕ = 40 μm. In models AC, a dot of ϕ = 20 μm, surrounded by a ring of ϕ = 150 μm was used. In total, five versions for each of the 2 models were obtained. Observation of a model consisted of the measurement of each model point, followed by the monocular measurement of the closest reseau point. Since the signalised model points are situated in groups of three, the same reseau point is measured three times.

Table 7 – Summary of available variants of each model

Abbreviation	Type of photography
GL	Diapositive on glass
FI	Diapositive on film
AA	Diapositives on film, exposed to a temperature of 30° C during approx. 15 minutes
AB	Diapositives on film, exposed to a temperature of 40° C during approx. 45 minutes
AC	Diapositives on film; Photos 39, 43 and 48 were treated differently: Photo 39: Washed during 1 night; dried at 20° C Photo 43: In fixing bath for 1 night; washing 1 hr; dried at 25° C Photo 48: Washed for 1 hr; dried at 40° C

?.1. Computation

As before, the basic data for each model consists of a list of image coordinates in the comparator system. Four reseau points in the corners of the photographs were used as fiducial marks and the comparator coordinates were affinely transformed into the fiducial system with the help of only these 4 reseau points.

Three sets of data were now obtained: one set without reseau points, one including all measurements of reseau points and the third including only one measurement per observed reseau point. This latter data set was obtained by deleting the second and third observation at each reseau point.

Table 8 summarises the magnitude of the film deformations in the observed reseau points, from which it can be concluded that the aging procedures did not deteriorate the geometric quality of the films to that extent as might have been expected.

Table 8 – R. m. s. value of film deformation in μm at photoscale. Values obtained in model 39/43 from 21, in model 43/48 from 28 reseau points, after affine transformation of measurements into the system defined by 4 fiducials

Photo	39		43	
	x	y	x	y
GL	5	4	8	5
FI	6	4	9	7
AA	4	2	12	5
AB	7	8	7	8
AC	4	5	11	8

(a) Model 39/43

Photo	43		48	
	x	y	x	y
GL	6	6	9	8
FI	7	6	11	8
AA	5	5	10	13
AB	7	12	16	13
AC	7	6	11	12

(b) Model 43/48

Next, the known deformations in the reseau points were used to interpolate corrections in the other points. Following the idea of *Kraus*, for this purpose a programme using linear prediction (least-squares interpolation) was written.

The correlation function in this programme can either be computed from the known film deformations in a subroutine, or a pre-specified function can be entered. Basically, the photo is split in several zones in such a way that not more than 30 reseau points are used for an interpolation.

Reseau points from adjacent zones are included in interpolations of a zone, if these reseau points are within a specified distance from the boundary line. This is to assure continuity of the interpolated corrections. The separation into zones is necessary to avoid excessively large correlation matrices.

Within each zone a trend polynomial of specifiable order is computed (with one to ten coefficients). The residuals left after the fit of the polynomial to the Δx and Δy deformations are used as input to the linear prediction.

The correlation functions used are of the isotropic type:

$$\text{Corr.} = C_0 \,/\, (1 + d^2 \,/\, C_1^{\,2}) \qquad (1)$$

where d is the distance between 2 correlated points.

This function has proved to be successful in many applications, e. g. interpolation of gravity anomalies, and in DTMs.

Attempts failed to compute the coefficients C_0 and C_1 from the residuals per zone. Presumably the number of reseau points was too small per zone. Use of a function computed for the whole of the photograph, or even from a part of it, has been proposed by *Kraus* (Z. Vermess.-wes., 1971/6). These functions are however not representative of the residuals left per zone after trend subtraction ("centering"). Therefore an optimum correlation function was experimentally found by "trial and error"; different functions were tried out, and the best one chosen.

Interpolation of Δx and Δy corrections has been done independently. Although the programme allows for the consideration of cross correlation between Δx and Δy, the simultaneous interpolation of Δx, Δy did not bring any improvement and so it was excluded from further consideration.

All three sets of image coordinates – without, with one measurement, with three measurements per observed reseau point – were used for relative and absolute orientation, applying the maximum number of available points as orientation points.

Relative and absolute orientation were carried out with more than 50 points in the case model 39/43, and with more than 70 points in the case of model 43/48. The residuals left after absolute orientation were divided by the scale, and represent the outcome of the experiment. This is discussed next.

4.2.2. Discussion of Results

The first step was the optimization of correlation function (1). Table 9 summarises the results obtained. Coefficients C_0, C_1 were varied systematically in the interpolation. Figure 9 shows graphical representations of some of the alternatives which were tried out. The values for C_0, C_1 providing the smallest r. m. s. residuals in the 3 coordinate directions were found to be

$$C_0 \approx .50 - .70$$
$$C_1 \approx 50 - 70 \text{ mm}$$

Table 9 – Experimental optimization of correlation function $C_0/(1 + d^2/C_1{}^2)$. Trend polynomial of 2^{nd} order (6 unknowns). Values in μm at photoscale

C_1 (mm)		10			30			50			70		
R. m. s. Value		ΔX	ΔY	ΔH	ΔX	ΔY	ΔH	ΔX	ΔY	ΔH	ΔX	ΔY	ΔH
C_0	1	5.3	4.1	9.2	7.4	5.8	12.1	38.3	29.8	44.8			
	.85	4.7	4.3	9.2	4.6	4.3	9.1	4.6	4.2	8.9	4.5	4.2	8.9
	.70	4.6	4.2	9.2	4.5	4.2	9.0	4.5	4.2	8.9	4.5	4.2	9.0
	.50							4.5	4.3	9.1	4.5	4.3	9.1

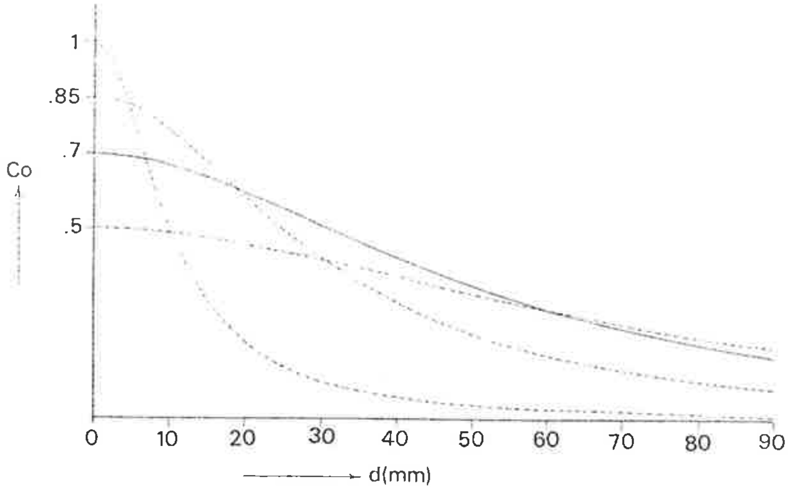


Figure 9 — A sample of correlation functions tested. The thicker line denotes the function (C_0 = .7, C_1 = 50) that gave the optimum result

Table 9 is computed from model AA 43/48, using one measurement per observed reseau point. Similar computations were carried out for other models, providing the same conclusion. The trend polynomial was chosen to be of 2^{nd} order (6 unknowns).

Taking the optimum correlation function, the number of unknowns of the trend polynomial was varied. The results are shown in table 10. Obviously there is no point in choosing more than a 2^{nd} order trend polynomial per zone.

Table 10 — Comparison of three trend polynomials, with 3, 6 and 10 coefficients respectively, for AC 43/48

		ΔX	ΔY	ΔH
No. of coef-ficients	3	4.7	4.1	10.0
	6	4.5	4.2	8.9
	10	4.6	4.4	8.9

In table 11, the r. m. s. values of the lack of coplanarity after relative orientation are given. Although there is a noticeable tendency for deteriorating stereoscopy with the aging of the photographs, the magnitude of differences is fairly small.

Table 11 – Lack of coplanarity after relative orientation, with and without use of reseau. Given are r. m. s. values in μm

Model	Reseau	GL	FI	AA	AB	AC
39/43	without	5.3	6.1	6.2	6.5	6.3
	with	5.9	6.0	6.6	6.5	5.7
43/48	without	5.4	5.8	6.7	7.6	7.0
	with	5.1	5.1	6.9	6.5	6.7

Table 12 – R. m. s. residuals ΔX, ΔY, ΔH in μm at photoscale, after absolute orientation. X-axis of terrain system approx. parallel with photo y-axis

	ΔX	ΔY	ΔH	$\Delta H/_Z$
GL	7.2	7.1	12.8	.084 ‰
FI	7.3	6.9	10.1	.066 ‰
AA	5.6	5.3	11.4	.074 ‰
AB	7.0	6.0	12.7	.083 ‰
AC	7.9	5.9	10.9	.072 ‰

(a) without reseau, 39/43

	ΔX	ΔY	ΔH	$\Delta H/_Z$
GL	4.6	5.5	9.8	.064 ‰
FI	5.5	5.2	9.8	.064 ‰
AA	4.4	5.0	9.6	.063 ‰
AB	4.5	4.8	8.9	.059 ‰
AC	4.1	4.1	10.0	.066 ‰

(b) with reseau, 39/43

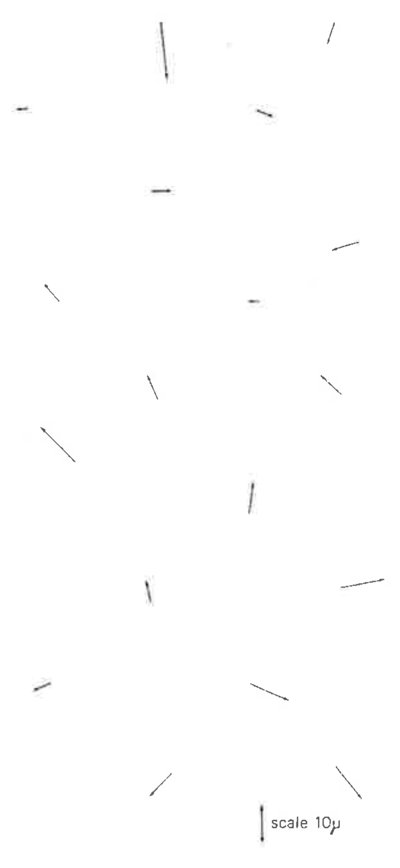
	ΔX	ΔY	ΔH	$\Delta H/_Z$
GL	6.7	6.8	12.4	.082
FI	5.2	5.9	13.3	.087
AA	5.9	6.0	12.3	.081
AB	7.1	5.8	14.1	.093
AC	6.1	4.8	15.5	.101

(c) without reseau, 43/48

	ΔX	ΔY	ΔH	$\Delta H/_Z$
GL	6.7	6.0	10.9	.072
FI	4.8	6.6	10.9	.072
AA	5.5	5.6	9.6	.063
AB	5.7	5.7	10.0	.066
AC	4.5	4.2	8.9	.059

(d) with reseau, 43/48

a) $\sqrt{\Delta x^2 + \Delta y^2}$ without reseau



b) ΔH without reseau

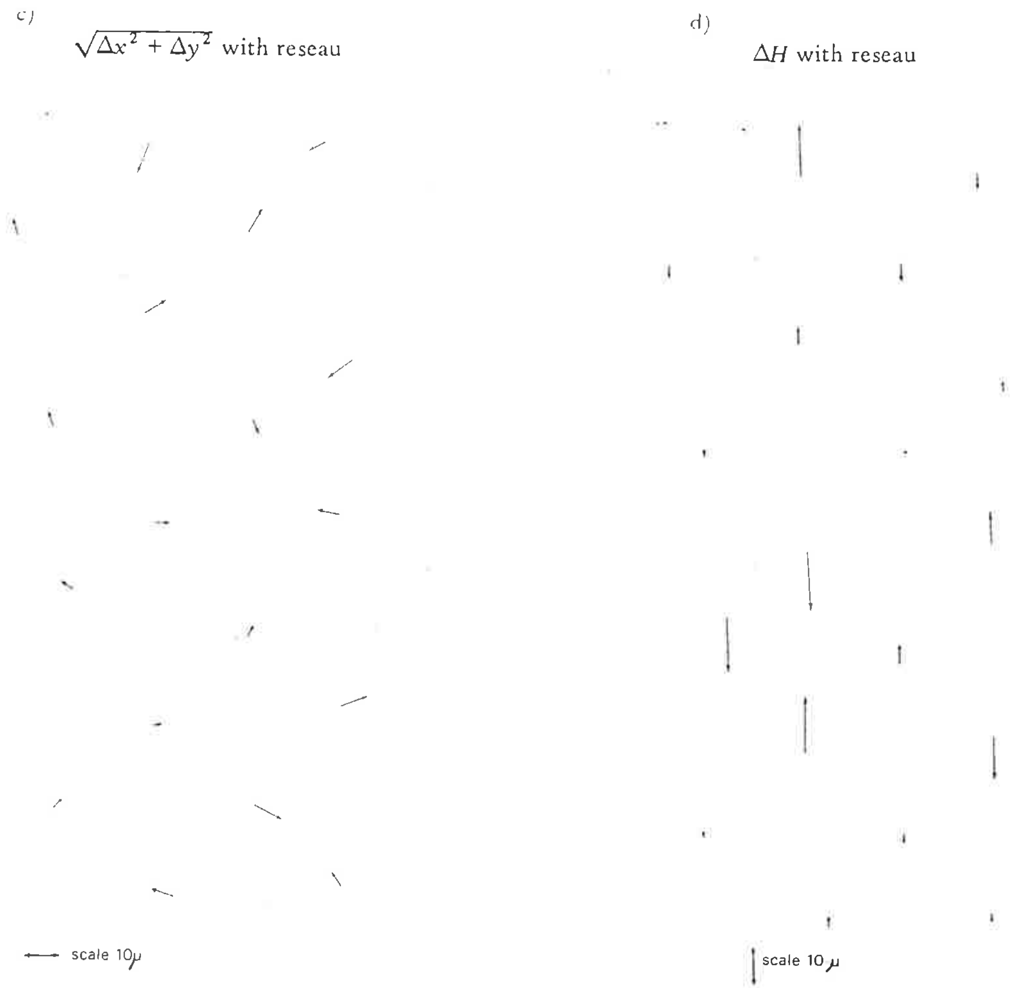


Figure 10 – Vector diagrams of residuals after abs. orientation, model 39/43, version AB

Table 12 shows the main result of the experiment, namely the r. m. s. residuals after absolute orientation of all 5 variants of each model, with and without use of the reseau. In all cases, except for *Y* of model FI 43/48, improvement could be obtained by the use of the reseau, with a maximum of almost a factor 2 for *H* in model AC 43/48.

This table refers to the case of a single measurement per observed reseau point. Almost identical results are obtained if all measurements of reseau points are included. Differences are only of the order of fractions of μm. Therefore, these results are not further discussed. There is no point in observing a reseau point more than once, if linear prediction is applied for interpolation.

For model 39/43, the use of reseau leads to a fairly uniform accuracy, whatever procedure of aging was taken. The results are close to the limit of the photogrammetric state of art. It is interesting to note, that no correction for lens distortion or refraction was applied, since the purpose of the experiment was only to demonstrate the use of the reseau. This in particular shows the geometric performance of the photography to be rather impressive.

In model 43/48, version AC gives results superior to glass or film diapositives. This does not justify the conclusion that artificial aging should become a standard photogrammetric procedure. The superiority of the results is purely due to the fact that a different floating mark was used, which allows especially planimetry to be measured with a higher accuracy. This effect is more pronounced in model 43/48 than in model 39/43, since the signalisation of control points causes problems especially in the former model.

For the case of model AB 39/43, a graphical plot of planimetric and height residuals was prepared. Figures 10a, b show the error vectors for planimetry and height without, and figures 10c, d with use of the reseau. The distribution of errors is clearly the same, but their magnitude is reduced by the reseau.

Finally, table 13 lists the maximum residual per case and coordinate direction, with and without reseau. These maxima are reduced by the reseau, however to a much smaller extent than the r. m. s. residuals.

Table 13 — Extreme residual after absolute orientation, without and with use of reseau, for model 43/48. Values in μm at photoscale

	ΔX		ΔY		ΔH	
	without	with	without	with	without	with
CL	– 25	– 22	– 18	– 18	– 32	– 32
FI	13	12	19	– 18	– 40	– 31
AA	16	– 14	18	– 17	– 32	– 25
AB	– 18	– 19	11	– 12	– 42	– 29
AC	– 14	10	17	– 11	– 42	– 25

4.2.3. Conclusion

The above experiment clearly demonstrates that the use of the reseau provides a uniform and excellent accuracy, irrespective of the film deformation introduced by artificial aging procedures. Without the use of reseau, the results were far more inhomogeneous from one version to another than with the reseau. The differences of the results between different aging methods, and between glass and film, are reduced to fractions of microns if reseau corrections are applied.

The absolute magnitude of the film deformations in the particular case of the two models under study was rather small. Post-processing of the developed films did not deteriorate the geometric quality to that extent as expected or hoped, so that a demonstration of the usefulness of reseau with "poor quality" photography was not spectacular. However, it could be clearly proved that the reseau is indispensable if the ultimate potential of the photogrammetric process is to be exploited.

4.3. Monocular Investigations

extension of the monocular reseau investigations was concentrated on two aspects, namely the influence of film aging and film storage on the geometric stability and a comparison of the stability of glass and film diapositives.

To this end, 15 diapositives on both glass and polyester film were printed from the original film negatives. These sets of 15 positives were selected at the beginning, middle and end of each of the 5 reseau strips. The three glass and film positives of the Rheidt test area were, of course, also included in the investigation, as were the "artificially aged" film positives.

The measurements were restricted to the 81 point grid in each plate, these then being conformally transformed using all points.

The following main conclusions could be drawn from the results:

- There was no significant difference in either the pattern or the magnitude of the distortion of the 15 "new" glass diapositives compared to those prepared some three years earlier.
- The differences between the residual distortions of the sets of glass and film diapositives were insignificantly small i. e. in the order of the pointing accuracy.
- The attempts to "artificially age" the film positives by alternately heating and soaking iese were unsuccessful.

In general, the conclusion reached from this material was that the deformation of the original film negatives was not adversely affected by storage and that diapositives on polyester film are certainly not inferior to those on glass as far as geometric stability is concerned.

RÉSUMÉ:

Les bandes 6, 8, 10, 12 et 14 du bloc grand'angulaire d'Oberschwaben ont également été photographiées avec la chambre à réseau de Zeiss (chambre RMK AR 15/23). A l'ITC, 25 diapositives sur verre de la bande 10, et 3 de chacune des bandes 6, 8, 12 et 14, ont été mesurées monoculairement sur un comparateur (voir figures 1–6). Par bande, les résultats étaient presque identiques sauf pour un «jeu» de 2.5 à 3 microns en x et y de chaque point de réseau. Ce jeu correspond pratiquement à l'erreur de mesure. Pourtant, pour les bandes différentes, les résultats étaient légèrement différents d'où l'on pourrait constater un systématisme significatif différent entre les bandes extrêmes 6 et 14.

Pour décrire les erreurs systématiques par photographie, ou par bande, ont peut diviser la photographie en sections, mesurer un nombre de points, et calculer l'erreur par section (voir figure 7). Pour la photographie à réseau d'Oberschwaben, le choix de l'une quelconque des 9 combinaisons (nombre de sections, nombre de points mesurés) révélait pratiquement aucune différence pour le but fixé. Une possibilité de correction pour les erreurs systématiques (dues à la déformation du film et à la non-planiété de la couche à l'instant de l'exposition) serait la suivante: mesurer – monoculairement – trois diapositives, une au début, une au milieu et une à la fin de chaque bande. Selon l'analyse de la figure 7, 25 points par diapositive suffiront. Si les trois diapositives montrent une distortion uniforme – comme c'etait le cas pour Oberschwaben – on pourrait corriger toutes les diapositives de la bande de ce systématisme. Si non, il faudrait mesurer un nombre approprié de points de réseau dans chaque diapositive individuelle.

Parallèlement avec l'investigation monoculaire, un test sur stéréorestitution de photographies à réseau a été exécuté dans lequel, après chaque observation sur un point stéréoscopique, on a mesuré le point de réseau le plus proche, à gauche et à droite, sur un stéréocomparateur. Chaque couple a été calculé deux fois, une fois en utilisant les observations des points de réseau (où la déviation constatée dans le point de réseau le plus proche est utilisée comme correction au point stéréoscopique) et une fois sans utilisation des observations des points de réseau.

Pour la triangulation de quelques bandes, après compensation selon *Ackermann,* les résultats, indiqués dans les tables 3 et 4, ont été trouvés: une précision très grande, indépendante de l'utilisation ou de la non-utilisation des observations des points de réseau.

Aussi pour la restitution de deux couples couvrant le champ d'essai de Rheidt (avec un grand nombre de points de contrôle signalisés) on n'a pas pu constater une différence significative entre les deux cas: utilisation ou non-utilisation des observations des points de réseau.

K. Kraus (Stuttgart), cependant, en appliquant une autre méthode pour la correction des points stéréoscopiques (c. a. d. une correction pas seulement basée sur la déviation constatée dans le point de réseau le plus proche, mais sur les déviations dans tous les points de réseau mesurés: «méthode d'interpolation linéaire selon le principe des moindres carrées») a trouvé un résultat un peu plus favorable (voir la table 6).

Finalement, à l'ITC, les observations ont été répétées avec des diapositives sur film, aussi que sur verre. Les diapositives sur film ont été «mal-traitées» afin de créer, artificiellement, un matériel moins bon que le matériel photographique original d'Oberschwaben. Ceci pour démontrer la valeur de la photographie à réseau.

Discussion about the Subject 11 under the Chairmanship of Prof. E. Gotthardt

Chairman: Thank you for your presentation Mr. *Visser*. Who would like to open the discussion?

Amer: I would like to know which strip adjustment procedure was used since this is of importance when comparing adjustments carried out with and without the use of the reseau?

Visser: We used the *Ackermann* strip adjustment procedure of 1961, which uses piece-wise polynomials.

Kupfer: Mr. *Visser* comes to the conclusion that the fiducial marks are not suited to correct the film deformation but I think this has to be proved in each flight.

Chairman: Can Prof. *Kupfer* show us that better results are obtainable from non-reseau cameras?

isser: I consider the reseau camera to be as good as any other.

Kupfer: So far we have only examined non-reseau cameras but now intend investigating the reseau camera.

Ligterink: Have you computed the variance factor for longer distances?

Visser: Dr. *Leberl* has so far used distances of up to 9 cm in the plate.

Förstner: What does S represent in fig. 8 of your paper?

Visser: S is the distance between points used to determine the covariance function. The example shows the y-component but a similar graph is also prepared for the x-component.

Ackermann: What is the effect of assuming a negligible cross correlation and treating the x and y corrections separately?

Leberl: In order to determine this we assumed a certain amount of cross correlation in one instance but the results were terrible. The problem, however, is one of interpolation and it is quite possible that polynomials would give a better result.

iairman: I have always had some doubts about the least squares interpolation procedure.

Kupfer: If there is a systematic trend present I would consider it advisable to start with regression polynomials since these would give a better result than with a least squares interpolation.

Leberl: I consider that we should not do complete injustice to the problem of linear prediction although I admit that it does depend on the experience and intuition of the user, since it is often difficult to determine the covariance function.

Visser: I would still advocate the use of the reseau to correct the systematic effects of film deformation.

Leberl: The effect of film deformation is extremely small.

Jerie: There are a number of other error sources which cannot be eliminated by means of a reseau.

Förstner: Mr. *Visser* claims that the reseau will not only control the effect of film distortion but also that of a poor flattening of the emulsion and of the photographic processing.

Visser: The reseau does however not control atmospheric influences such as refraction and air turbulence.

Holm: If I remember correctly, the photography was flown on 8 different days and so could not this be used to determine the influence of atmospheric conditions?

Furthermore, could not the different film types used be included in the investigation?

Kupfer: I am unaware of the fact that different film types were used in photographing the Rheidt test area.

Ackermann: The different film types used refer to the photography of the Oberschwaben block, but no double coverages were flown. The fact that the flying took a total of 8 days spread over a period of two months is largely due to poor weather conditions and due to the difficulties encountered in obtaining pin-point photography.

Chairman: I am afraid that I have to close this session now and I want to thank you all for your contributions.