INSTITUTE OF NAVIGATION AND SATELLITE GEODESY GRAZ UNIVERSITY OF TECHNOLOGY
DIIAR POLITECNICO DI MILANO

Geoid Determination Combining Ground Data and Satellite-derived Global Models

R. Pail, N. Kühtreiber, B. Wiesenhofer
M. Reguzzoni, F. Sansò

Introduction

The combination of terrestrial data and satellite-derived global models can be interpreted in two ways:

1) The incorporation of satellite-related data from the dedicated gravity field missions CHAMP and GRACE (and in the future GOCE) into local and regional geoid solutions will stabilize and thus considerably improve them, particularly concerning its medium and long wavelength content and by reducing systematic effects such as biases and tilts.

2) Global satellite-only models are complemented by local data in order to improve the spatial resolution of the global model, or to reduce specific weaknesses of the global models, e.g., the polar data gaps of global models derived from the satellite mission GOCE.

In this poster several adaptations of the Least Squares Collocation (LSC) method to compute an optimum joint solution from local (terrestrial) and global data shall be presented, with special emphasis on the involved functional and stochastic models, and the choice of a consistent covariance function.

A collocation solution to combine global and local gravity data

In order to predict any gravity field functional z, the optimal linear combination between a set of spherical harmonic coefficients s_0 and local observations such as gravity anomalies Δg_0 can be formulated by LSC as:

$$\hat{z} = \lambda^T s_0 + \mu^T \Delta g_0 = \left[C_{z,s} \;\middle|\; C_{z,\Delta g} \right] \cdot \begin{bmatrix} K+\Sigma_s & C_{s,\Delta g} \\ C_{\Delta g,s} & C_{\Delta g,\Delta g}+D \end{bmatrix}^{-1} \cdot \begin{bmatrix} s_0 \\ \Delta g_0 \end{bmatrix} \qquad (1)$$

- K is the diagonal covariance matrix of the spherical harmonic coefficients, depending only on the degree variances (truncated at a maximum degree),
- Σ_s is the generally full error covariance matrix of the global model,
- $C_{i,j}$ is the signal (cross) covariance matrix between the functionals i and j,
- D is the noise covariance matrix of the local observations.

Due to large correlations among the local μ and global λ components, this approach fails in practical application. On the other hand, the same solution can be written in a more convenient way by exploiting the consistency among the covariance functions of the involved gravity field functionals. The normal equations related to Eq. (1) read:

$$KB_z^T = (K+\Sigma_s)\lambda + KB_{\Delta g}^T\mu = \bar{K}\lambda + KB_{\Delta g}^T\mu \qquad (2)$$

$$C_{\Delta g,z} = B_{\Delta g}K\lambda + (C_{\Delta g,\Delta g}+D)\mu = B_{\Delta g}K\lambda + \bar{C}_{\Delta g,\Delta g}\mu \qquad (3)$$

where B_i are the base functions related to the gravity field functional i. Solving Eq. (2) with respect to λ and inserting the result into Eq. (3) yields the normal equations of μ in the reduced form:

$$\left[C_{\Delta g,z} - B_{\Delta g}K\bar{K}^{-1}KB_z^T\right] = \left[\bar{C}_{\Delta g,\Delta g} - B_{\Delta g}K\bar{K}^{-1}KB_{\Delta g}^T\right]\mu \qquad (4)$$

Improvement of local geoid

In this case the functional z to be predicted is the geoid height N. An extended remove-restore strategy, which also includes a correct error description of the global component, is formulated. The coefficients μ are derived from Eq. (4) and then the final estimate of the geoid height reads:

$$\hat{N} = B_N K\bar{K}^{-1}s_0 + \mu^T\left[\Delta g_0 - B_{\Delta g}K\bar{K}^{-1}s_0\right] \qquad (5)$$

Numerical case study

Test Data Environment

* Simulated gravity anomalies Δg_0 based on a global gravity field model complete to D/O 1800, defined at 5796 irregularly distributed stations in Austria and its neighbourhood, superposed by random noise with $\sigma_{\Delta g} = 1$ mgal (*Fig. 1*).
* Spherical harmonic coefficients s_0 of the same global model complete to D/O 80, together with a realistic full covariance matrix Σ_s.

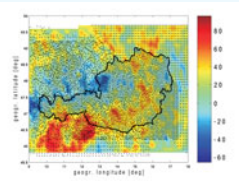

Fig.1: Grav. anom. Δg [mGal] at 5798 stations (black dots).

Simulation Results

Fig. 2 shows the resulting geoid height differences (left) and error estimates (right) of the local-only solution based on the Δg data, while *Fig. 3* illustrates the solution based on global and local data, where the error estimation includes a correct propagation also of the global gravity model errors described by Σ_s.

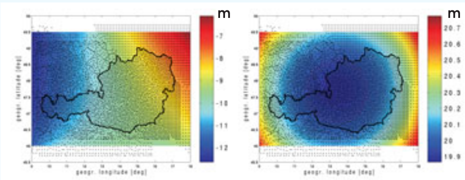

Fig.2: Geoid height [m] differences (left) and error estimates (right) based on local Δg data: Insufficient representation of long-wavelength gravity field information leads to long-wavelength errors in the order of 10 m.

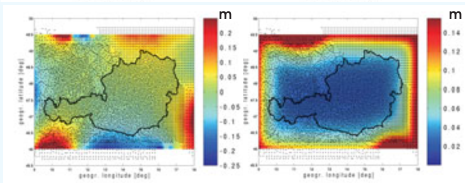

Fig.3: Geoid height [m] differences (left) and error estimates (right) based on local and global data: By inclusion of a global gravity field model the long-wavelength errors could be considerably reduced.

Improvement of global geoid

In this case the functional z to be predicted is an updated version of the spherical harmonic coefficients s. In the simplest case of model refinement without increasing the spatial resolution (i.e. $B_z = I$), the coefficients μ are derived from Eq. (4) and the optimal data combination results:

$$\hat{s} = K\bar{K}^{-1}s_0 + \mu^T\left[\Delta g_0 - B_{\Delta g}K\bar{K}^{-1}s_0\right] \qquad (6)$$

Numerical case study

Test Data Environment

* Gravitational potential is simulated on a spherical grid at GOCE satellite altitude (250 km) from EGM96 up to degree/order 90 (see *Fig. 1*).
* White noise is added with a variance depending on the latitude (satellite data have higher density close to the poles and lower close to the equator).
* Polar gaps are simulated by adding noise with a much higher variance above latitude 83°.

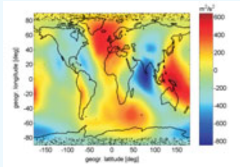

Fig.1: Grav. potential [m^2/s^2] at satellite altitude (250 km). 1000 ground data (black dots).

Simulation Results

* Spherical harmonic coefficients s_0 are estimated from the simulated grid data along with the error covariance matrix, which has a block-diagonal structure. Polar gaps produce a bad estimate of low-order coefficients (see *Fig. 2*).
* 1000 observations of gravity anomalies Δg_0 at ground level are simulated over latitude 80° with a white noise of 1 mgal (see *Fig. 1*). A new global model is re-estimated by combining the previously computed model and local data. The effect of polar gaps is now compensated (see *Fig. 3* and *Tab. 1*).

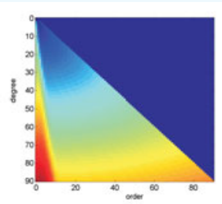

Fig.2: Estimation error by using satellite data only.

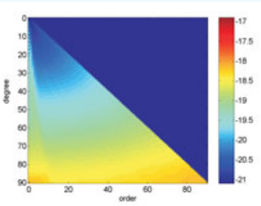

Fig.3: Estimation error after improving the global model by ground polar data.

model	geoid	grav. anom.
original	1.27 m	14.5 mgal
updated	0.37 m	3.9 mgal

Tab.1: Commission error up to degree/order 90 and for $-90° < \varphi < 90°$.

Acknowledgments

This work was performed in the frame of the FFG-funded project "The Austrian Geoid 2007 (GEOnAUT)" of the Austrian Space Application Programme (ASAP).

Conclusions

In this poster the problem of the combination of local (terrestrial) gravity field data and global gravity field information is addressed. The main simplification with respect to reality is that the global and local data have been simulated based on a consistent covariance model. Future work will address the necessary adaption of the covariance model to local data, and a still consistently joint solution of the combined system.