

MODELLING CABLE DYNAMICS EXEMPLIFIED BY LOAD TRANSPOSITION OF INSULATOR STRINGS IN OVERHEAD LINES

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Abstract: *This paper outlines perspectives and different approaches for describing mechanical behavior of cable dynamics. The characteristics of insulator strings in overhead lines during load transposition is important for a save and fast dimensioning process. Conclusive simulation results are rooted in an adequate model of the conductor cable. In this paper linear and nonlinear cable models, based on the FEM approach are compared to analytical and numerical models based on the equation of vibrating string. This comparison of models with different levels of complexity provides valuable insights on the quality of results (in this case the force response of cable at the end). All models are implemented, validated by bench tests and coupled with a Multi Body Simulation model of the falling insulator string. Results are shown and ratings of all presented models are provided. In addition to the problem of load transposition in overhead lines, this paper gives advices on simulating cable dynamics in other fields of applications like structural engineering, materials handling or mechanical engineering. Thus the paper can help the reader to choose the right cable model for simulating a particular problem.*

Key words: *cable dynamics, load transposition, equation of vibrating string, FEM, overhead line, insulator string, sag flat cable.*

1. INTRODUCTION

Cables and ropes are essential for various applications in engineering. For the design and development of technologically advanced systems affected by the behavior of cables, extensive analysis and simulations are mandatory. Otherwise securing functions in terms of external influences (external forces, excitation...) is not ensured.

There are two major categories for classification of cables. One is the type of design; the other is the field of application. Figure 1 provides an overview of the most common cable types. The dimensioning process by forming a quotient of tensile stress at break and the tension stress is attended by high coefficients of safety (3 to 14 see [1, 2]). A reason is the difficult calculation of stresses in cables under dynamic conditions. Inaccurate approaches using static calculations and various factors for considering dynamic effects are established. Detailed simulations of the dynamic behavior of cables can provide more accurate results (tensile stress). A numerousness of approaches is known for modelling cable dynamics. This paper gives an overview and demonstrates the impact of different cable models using the example of load transposition (LT) of overhead lines.

Load transposition Process. For insulating the conductors from environment, insulator strings are placed between conductor and pylon. To avoid black-out or mal-

function in case of insulator breakage, at least double strings are often used. The reasons for breakage are material defects, falling rocks, vandalism and branches of trees. The design, using double strings, ensures that the intact insulator string take over the whole conductor load.

The process of taking a new equilibrium is called load transposition and lasts only about 0.3 seconds. High bending stresses in the brittle porcelain insulators caused by the angular acceleration of the insulators are characteristic for the process. For a proper dimensioning of the insulator strings, numerical simulations (with adequate cable models) are very helpful [1, 2].

2. APPROACHES FOR CABLE DYNAMICS

The approaches for modelling the dynamics of sag flat cables, found in literature, are varying concerning their complexness. Figure 2 provides an overview of approaches, classified in analytical and numerical methods referring to cable properties, excitation, nonlinear effects and attainable results. Depending on the field of application (Fig. 1) and effects to be considered, an adequate approach can be chosen with the help of Table 1.

The next chapters are describing two different approaches with different complexity in detail. The first approach, based on the equation of the vibrating string, is able to deliver results very quick. The second approach, based on non-linear FEM, is more sophisticated and can handle miscellaneous effects. Also the effort for implementation and computing times are varying. The impact using different approaches are exemplified by load transposition of overhead lines (chapter 4).

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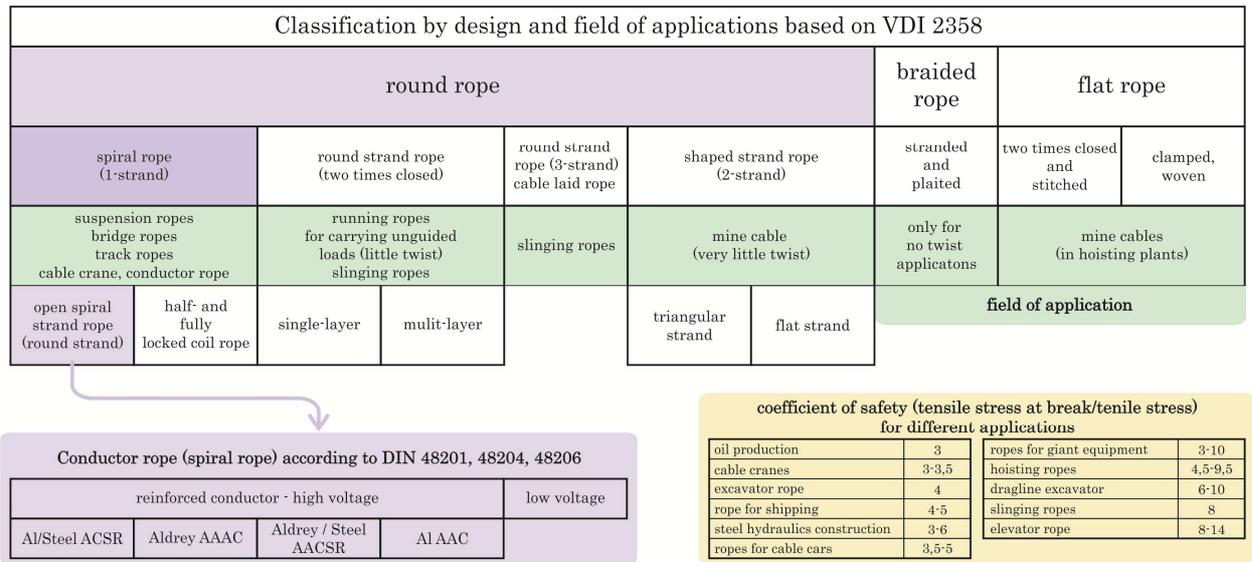


Fig. 1. Classification of cables by design and field of applications (according to VDI 2358).

Table 1

Overview - approaches for cable dynamics

| | Finite Elemente Methoden | | | weitere numerische Methoden | | analytische Methoden | |
|--|--|-----------------|--------------|--|---|---|--|
| | Balkenelemente | krumml.Elemente | Stabelemente | diskrete Modelle aus Federn, Massen und Dämpfern | vereinfachte Abb. als endlich lange Saite | vereinfachte Abb. als unendlich lange Saite | Wellengl. auf Basis der parabolischen Seilline |
| Berücksichtigung von: | ← Tendenz zunehmender Genauigkeit / Rechenzeit → | | | | | | |
| Seildurchhang | ja | ja | ja | ja | nein | nein | vereinfacht |
| Biegesteifigkeit des Seils | ja | je nach Element | nein | möglich (Drehfedern) | nein | nein | nein |
| Torsionssteifigkeit des Seils | ja | nein | nein | möglich (Drehfedern) | nein | nein | nein |
| Schubsteifigkeit Seils | ja | nein | nein | nein | nein | nein | nein |
| Stetige Abbildung des Seils | ja | ja | nein | nein | ja (gespannte Saite) | ja (gespannte Saite) | ja |
| Reflexionen am Seilende (z.B. an Masten) | ja | ja | ja | ja | ja | nein | ja |
| nicht periodischer Erregung | ja | ja | ja | ja | ja | ja | nein |
| hochfrequenter Erregung | ja, bei ausreichend hoher Diskretisierung | | | ja, bei ausreichend hoher Diskretisierung | | ja | bedingt (durch Wahl entspr. Eigenfrq.) |
| Erregung durch diskrete Ereignisse (Eisabwurf) | ja | ja | ja | ja | möglich (aber aufwendig) | nein | nein |
| Auslenkungen quer zur Durchhangebene | ja**** | ja**** | ja**** | ja | teilweise | teilweise | teilweise |
| großen Verschiebungen u. Winkeländerungen | ja**** | ja**** | ja**** | ja | nein | nein | nein |
| großen Dehnungen (geom. nichtlinear) | ja*** | ja*** | ja*** | nein | nein | nein | nein |
| nichtlinearer Seil E-Modul | ja** | ja** | ja** | nein | möglich (aber aufwendig) | nein | nein |
| Dyn. Kräfte rückwirkung infolge Selbsterregung | ja | ja | ja | ja | ja | ja | ja (nur stationäre Zustände) |
| Dyn. Kräfte rückwirkung infolge Auflagerbewegung | ja | ja | ja | ja | ja | ja | ja (nur stationäre Zustände) |
| Schwingungsverhalten des Seils | ja | ja | ja | ja | vereinfacht | vereinfacht | ja (nur stationäre Zustände) |

| |
|--|
| Erforderliche Stufe n.l. FEM Rechnung: * linear ** nur materiell nichtlinear *** mat. und geo. nichtlinear (K nicht aktualisiert) **** mat. und geo. nichtlinear (K aktualisiert) |
|--|

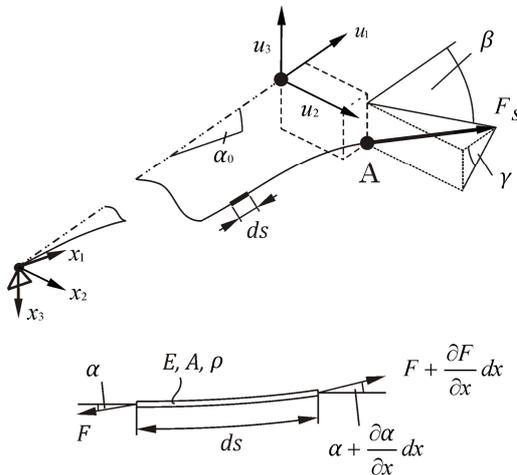


Fig. 2. The vibrating string.

2.1. Equation of Vibrating String

An approach for modelling cable dynamics is using the equation of vibrating string (first efforts calculating the vibrations of strings were made in the 17th century). For solving the equation, analytical and numerical methods are possible [5].

The equations of motion, obtained from Fig. 2, are leading to a partial differential equation of second order with two independent variables x and t (position and time). Thus the basic equation for the vibrating string (small angular displacements) is [5]:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}. \quad (1)$$

The variable c is the longitudinal wave velocity (index l_0) respectively the transversal wave velocity (index t_r) and u the displacement of cable.

$$c_{l_0} = \sqrt{\frac{E}{\rho}}; \quad c_{t_r} = \sqrt{\frac{F_0}{\rho A}}. \quad (2)$$

Typical values for conductor cables are 4.700 m/s (c_{l_0}) and 85 m/s (c_{t_r}).

The approach of the vibrating string underlies some simplifications (Table 1):

- Sag of cable is neglected.
- Bending stiffness is neglected.
- Linear material model.
- Constant density.

The following methods for solving of (1) are based on the assumption, that longitudinal and transversal vibrations are independent (superposition of waves is possible).

String equation (infinite length-analytical solution). Assuming an infinite string, a simplification of d'Alembert's solution leads to [6]

$$S_d = S_0 - \frac{EA}{c_L} \dot{u}_A, \quad (3)$$

where the dynamic force S_d in longitudinal direction of the cable depends on the velocity \dot{u}_A of point A (Fig. 4) in the direction of x_1 . The reader may notice that reflected waves (e.g. on neighbour pylons) are not considered. The numerical solution of (1) addresses this issue.

String equation (infinite length-numerical solution). Due to the fact that the characteristic of displacement at least at one end of the cable is unknown during simulation (the other end is supposed to be fixed), a numerical solution might be applied. Replacing the partial derivations in (1) by terms of Taylor series approximation leads to

$$u_j^{n+1} = 2(1 - \alpha^2)u_j^n + \alpha^2(u_{j+1}^n + u_{j-1}^n) - u_j^{n-1}, \quad (4)$$

where j indexes discrete points on the cable, n is the index of discrete time steps [7]. Equation (4) is stable under the Courant-Friedrichs-Lewy (CFL) condition [8]

$$\alpha = c_L (\Delta t / \Delta x_1), \quad (5)$$

Δt and Δx_1 are finite differences of time and cable.

With the displacements u_j^n and applying a central difference method the dynamic force S_d follows as:

$$S_d = (\Delta u / \Delta x_1) EA. \quad (6)$$

The impact of considering reflections will be discussed in chapter 4.

2.2. Non-linear FEM (2-Node Truss Elements)

In contrary to approaches based on the vibrating string, FEM is able to consider different effects such as discrete events, sag of cable, and so on (Table 1). FEM with truss elements is commonly used for analysing structures like bridges or buildings. But also modelling cable dynamics is possible (under the premise, that the curvature of cable remains relative small during simulation).

A truss element is a structural member capable of transmitting stresses only in the direction normal to the cross section. It is assumed that this normal stress is constant over the cross-sectional area. The element (length L) is described by two nodes (linear interpolation functions), as shown in Fig. 3 [9].

The 2-node truss element in Total Lagrange Formulation (TL) is suitable for large displacements, large rotations and small strains. In Lagrangian incremental analysis approach, the equilibrium of the body at time $t + \Delta t$ with reference to time $t = 0$ (index on the left below) expressed by using the principle of virtual displacements.

The general matrix equation for dynamic analysis with implicit time integration according the TL-formulation reads as follows [9]:

$$\mathbf{M}^{t+\Delta t} \ddot{\mathbf{u}} + \mathbf{C}^{t+\Delta t} \dot{\mathbf{u}} + \left({}^t \mathbf{K}_L + {}^t \mathbf{K}_{NL} \right) \Delta \mathbf{u} = {}^{t+\Delta t} \mathbf{R} - {}^t \mathbf{F}. \quad (7)$$

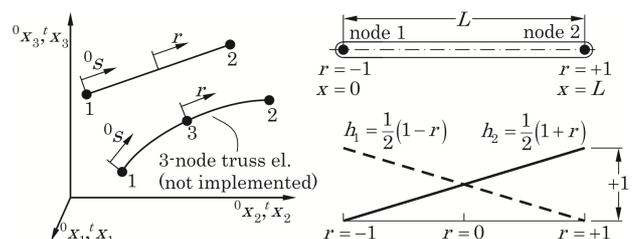


Fig. 3. Truss element with interpolation functions.

The finite element matrices in equation (7) necessary for nonlinear analysis are:

- Linear strain incremental stiffness matrix $\mathbf{K}_{L..}$
- Nonlinear strain (geometric) incremental stiffness matrix $\mathbf{K}_{NL..}$
- Time independent mass matrix $\mathbf{M}\mathbf{v}$.
- Time independent damping matrix $\mathbf{C}\mathbf{v}$.
- Vector of the externally applied nodal point loads $\mathbf{R}\mathbf{v}$.
- Vector of nodal point forces \mathbf{F} equivalent to the element stresses.

For solving (7), a nonlinear calculation scheme with an implicit time integration method is required. To meet these specifications, the Newton-Raphson method with time integration (trapezoidal rule) is applied.

The incremental formulation of the displacements, velocities and accelerations are (k is the number of the Newton iteration and Δt the size of time step):

$${}^{t+\Delta t}\mathbf{u}^{(k)} = {}^{t+\Delta t}\mathbf{u}^{(k-1)} + \Delta\mathbf{u}^{(k)}, \quad (8)$$

$${}^{t+\Delta t}\dot{\mathbf{u}}^{(k)} = \frac{2}{\Delta t} \left({}^{t+\Delta t}\mathbf{u}^{(k-1)} - {}^t\mathbf{u} + \Delta\mathbf{u}^{(k)} \right) - {}^t\dot{\mathbf{u}}, \quad (9)$$

$${}^{t+\Delta t}\ddot{\mathbf{u}}^{(k)} = \frac{4}{\Delta t^2} \left({}^{t+\Delta t}\mathbf{u}^{(k-1)} - {}^t\mathbf{u} + \Delta\mathbf{u}^{(k)} \right) - \frac{4}{\Delta t} {}^t\dot{\mathbf{u}} - {}^t\ddot{\mathbf{u}}. \quad (10)$$

Detailed information is provided in [9, 10].

3. VALIDATION OF APPROACHES

For validating different cable models, a test bed on the Institute of Logistics Engineering is used. With this

test bed, the general behavior (not the load transposition process) of the oscillating cable is in focus. The installation ensures a sinusoidal displacement of one end of the cable and the measure of the force response (fig. 3). The test bed is driven by a servomotor and a cam plate for creating a linear movement.

In Fig. 4 the results of the comparison of measurement and simulation for an excitation in x_3 direction are shown. The diagrams illustrate the force response of the cable in x_1 -direction.

The results of the non-linear simulation with 2-node truss elements have the best accordance with measurement. Amplitudes and also frequency spectrum (FFT-analysis in [4]) are matching.

On the other hand the calculated amplitudes, using the model of vibrating string (infinite), are too low. Main reasons for the big difference are:

- Sag of cable is neglected in this model.
- A linear analysis without actualization of cable coordinates.

The string in this model has infinite length (stiffness of cable in longitudinal direction is too low).

So for this special case, the simplified model of the vibrating string is not suitable. Nevertheless the situation during load transposition of overhead lines is very different. Mainly the span-length (approx. 300 m) is much longer than on the test bed (15 m). In Chapter 4 this problem is analyzed by comparing the impact of different cable models on the results of simulating the load transposition process.

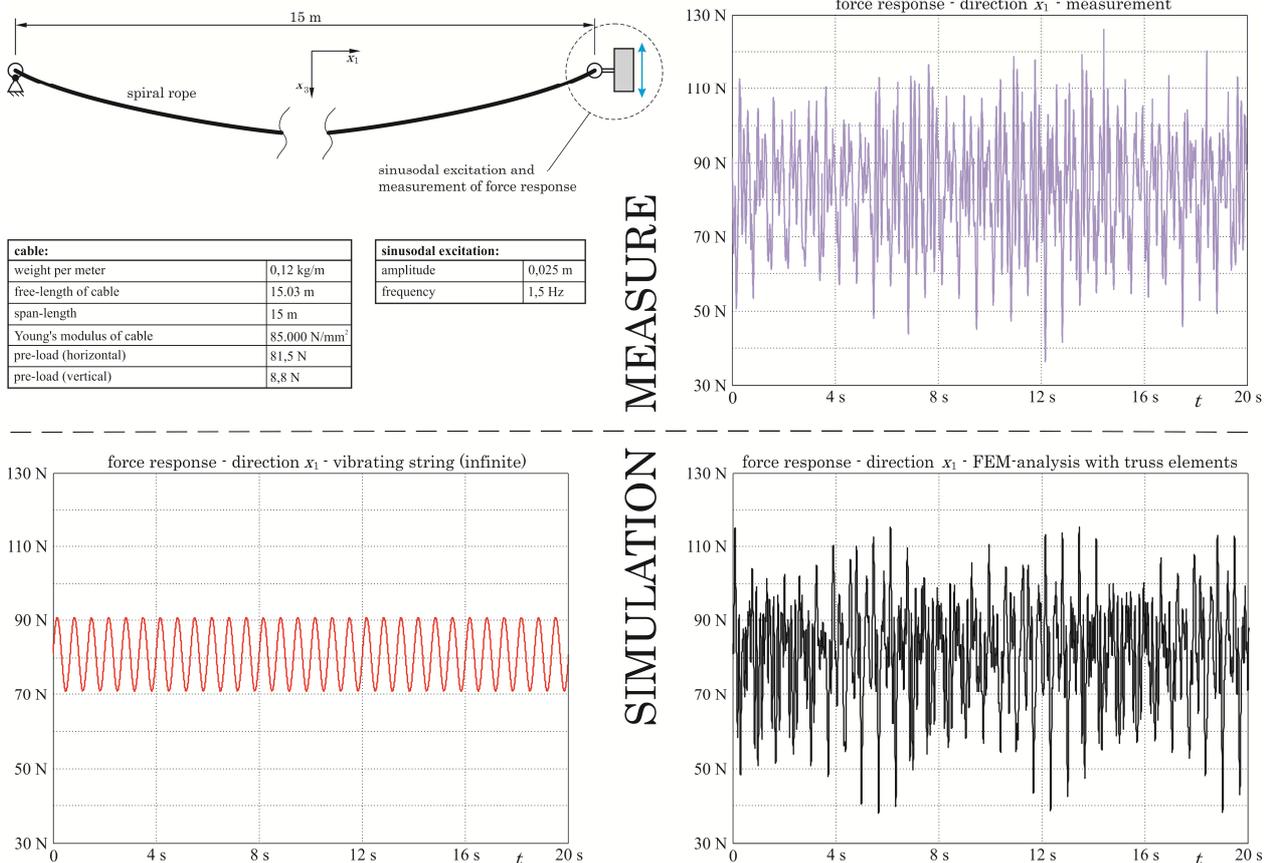


Fig. 4. Comparison of measurement and simulation.

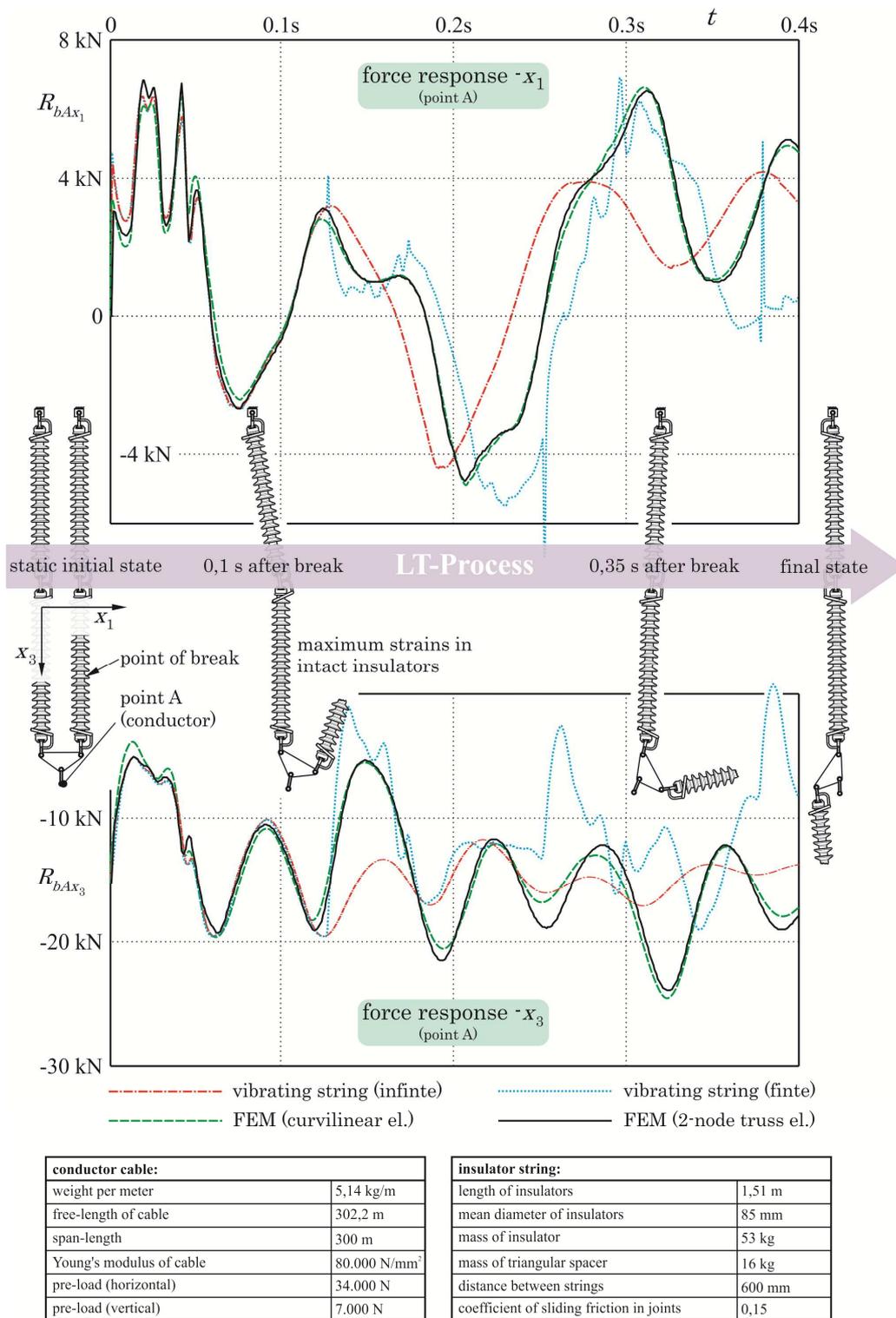


Fig. 5. Impact of different approaches on the simulation of the LT-process.

4. SIMULATION OF THE LT- PROCESS

Figure 5 shows the results simulating the LT-Process for a configuration with two insulator strings and a triangular spacer.

Up to 0.125 s after breakage of insulator, the difference between the force-plots on point A are very small. After this point in time, longitudinal wave reflections have to be considered. Due the fact that the model based on the infinite string doesn't considers reflections, the results achieved by using this approach are differing very

strong. On the contrary the approach of the vibrating string with finite length leads to very high force amplitudes (finite length and negligence of sag).

The purpose simulating the LT-process is the calculation of the maximum values of stresses and strains in the insulators. Since the maximum values always occur before the first reflections have an effect, all approaches can be used for simulating the LT-process. If a longer time frame is of interest (e.g. motion behavior of falling insulator strings), than a FEM-approach should be used.

Major points for modelling cables in LT-simulation are:

- Sag of cable has only little influence on maximum stress values. In the case of longer a simulation time, the sag has significantly more influence.
- A continues representation of cable (FEM with curvilinear elements [11]) has also only little influence on the results.
- In case of large lateral (horizontal) displacements of the cable during the LT-process, a nonlinear approach is highly recommended.

The models for simulating the LT-process are validated by measurements on overhead lines [3, 4, 6]. These validated models allow engineers a fast dimensioning process and the possibility to perform parameter studies.

5. SUMMARY AND OUTLOOK

The results shown in chapters 3 and 4 provide valuable insights in the modelling of cable dynamics. Not only for the special use case of load transposition, but also other applications benefit from the findings shown in this paper. Generally valid points for modelling cable dynamics are:

- The model of the vibrating string is only suitable for problems with abrupt and shock-type excitation. The simulation time should be short (because of the influence of wave velocity respectively wave reflections).
- For analysing a longer period of time, non-linear FEM approaches are the best choice. Very important is an actualization of nodal coordinates during the simulation (minor excitations can lead to relative large displacements of the cable).
- For simulating in-plane (no lateral cable movement) cable dynamics with small displacements also linear FE-Methods can be used. Linear models have the advantage of lower computing times, contrary to non-linear models.
- In case of using a FE-Method, an adequate numerical method for solving (time integration, nonlinear behavior) is very important.

In general, nonlinear methods lead to very good results. But they are suffering from high computing times. For that reason, it is necessary to prove which approach is suitable for different problems. This paper should give advices to take informed decisions.

A further objective is to apply the research results on other problems and fields of applications, including in particular the field of material handling. In logistics engineering, cables and belts are important machine parts. Using adequate approaches solving cable dynamics, promises a significant improvement in design processes.

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